2.3 FINDING ZEROS OF POLYNOMIALS

FACTOR THEOREM

A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$

Example 1: Use the factor theorem to show that the binomials are factors of the function.	
Show that $(x-2) and (x+3)$ are factors of $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$. $x-2 \Rightarrow f(2)$ $f(2) = 2(2)^4 + 7(2)^3 - 4(2)^2 - 27(2) - 18$ $f(2) = 0 \rightarrow$ so $x-2$ is a factor of $f(x)$ $x+3 \Rightarrow f(-3)$ $f(-3) = 0 \rightarrow$ so $x+3$ is a factor of $f(x)$	
Show that $(x+3)$ is a factor of $x^3 - 19x - 30 = 0$ $x+3 \Rightarrow f(-3)$ $f(-3) = (-3)^3 - 19(-3) - 30$ $f(-3) = 0 \rightarrow$ so $x+3$ is a factor	

To Find All Zeros of a Polynomial

1) Graph the function to find one of its roots/zeros.

2) Use synthetic division to simplify the polynomial.

3) Factor or use the quadratic formula to find all remaining roots/zeros.

Example 2: Find the real zeros of $f(x) = -2x^3 + x^2 + 10x - 5$ given that (2x -1) is a factor. Then sketch the graph.

zero

$$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

$$0 = -2x^3 + x^2 + 10x - 5$$

$$0 = (2x-1)(-2x^2+10)$$

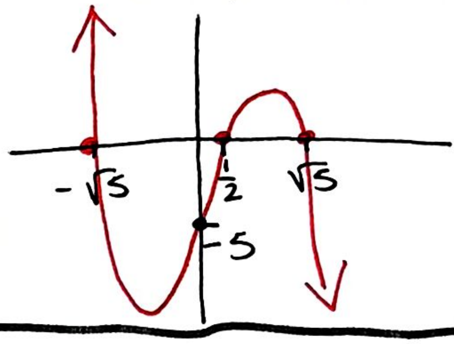
$$\begin{array}{c|cccc} & x^3 & & & \\ \frac{1}{2} & -2 & 1 & 10 & -5 \\ & \downarrow & -1 & 0 & 5 \\ \hline & -2 & 0 & 10 & 0 \\ & x^2 & x & \# & \end{array}$$

$$-2x^2 + 10 = 0$$

$$\frac{-2x^2}{-2} = \frac{-10}{-2}$$

$$\sqrt{x^2} = \sqrt{5}$$

$$\boxed{x = \frac{1}{2}} \qquad \boxed{x = \pm\sqrt{5}}$$



Quadratic Formula (for use on quadratics that cannot be factored)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3: Use your graphing calculator to find one zero of f(x), then find the exact values for the remaining zeros of the function.

$$f(x) = 2x^4 + x^3 - 4x^2 + x$$

$x = 1$ zero from graph

$$0 = 2x^4 + x^3 - 4x^2 + x$$

$$0 = x(2x^3 + x^2 - 4x + 1)$$

$$0 = x(x-1)\underbrace{(2x^2 + 3x - 1)}_{\text{use quad formula to find remaining zeros}}$$

$$\begin{array}{c|cccc} & x^3 & & & \\ 1 & 2 & 1 & -4 & 1 \\ & \downarrow & 2 & 3 & -1 \\ \hline & 2 & 3 & -1 & 0 \\ & x^2 & x & \# & \end{array}$$

Zeros at $\boxed{x = 0, 1, \frac{-3 \pm \sqrt{17}}{4}}$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$