2.6 Rational Functions and Asymptotes Notes

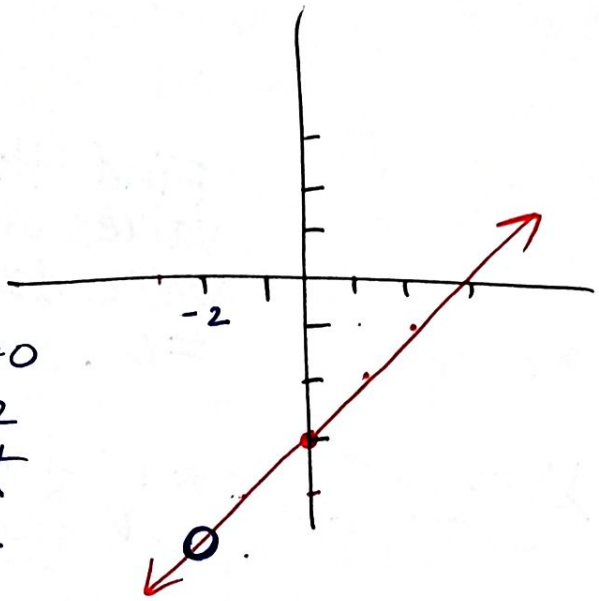
Asymptote	How to Find	Example
Vertical $x = \_\_\_\_$	Find the values of x that make denominator $= 0$. * Factor & set all factors $= 0$	$y = \frac{5x^2}{x^2-1} = 0$ $\rightarrow (x+1)(x-1) = 0$ $x+1=0 \quad x-1=0$ $x=-1 \quad x=1$ 2 vert. asymp.
Horizontal $y = \_\_\_\_$	Dominant Term (DT) = term(s) w/ the highest exponent in a function $\frac{DT}{DT}$ = horiz. asymptote $\frac{1}{DT}$ = horiz. asymp. at $y=0$ $\frac{DT}{1}$ = no H.A. or slant asymp. occurs	$y = \frac{5x^2}{x^2-1}$ H.A. $y = \frac{5x^2}{x^2} \Rightarrow y = 5$ $y = \frac{x^3-800}{4x^3+x^2+1000}$ $y = \frac{x^3}{4x^3} \Rightarrow y = \frac{1}{4}$
Slant $y = mx+b$	* only occur when the degree of the numerator is exactly 1 more than the degree of the denominator Divide the numerator by the denominator using synthetic division.	$y = \frac{2x^{(2)}}{x^{1}-1}$ ← degree = 2, ← degree = 1 $1 \mid 2 \quad 0 \quad 0$ (x^2, x, #) $\downarrow \quad 2 \quad 2$ $2 \quad 2 \quad 2$ (x, #, REM.) $2x+2+\frac{2}{x-1}$ $y = 2x+2$ slant asymp.

"Holey" Functions

$y = \frac{x^2 - x - 6}{x + 2}$

$y = \frac{(x-3)(x+2)}{x+2}$ ← $x+2=0$, $x=-2$

$y = x - 3$ hole at $x = -2$



*Holes in functions occur at x-values that make the original function undefined, but whose factors have canceled out.

Ex: Find all asymptotes and possible holes for $y = \frac{4x^2+24x+36}{x^2+4x+3}$. Deg = 2, Deg = 2 } same Degree

$y = \frac{4(x^2+6x+9)}{(x+1)(x+3)}$

$y = \frac{4(x+3)(x+3)}{(x+1)(x+3)}$ Hole at $x = -3$

H.A. ⇒ $y = \frac{4x^2}{x^2}$

$y = 4$

V.A. $x+1=0$

$x = -1$

Vertical Asymptote: $x = -1$
Horizontal Asymptote: $y = 4$
Slant Asymptote: none
Holes: $x = -3$