Trigonometry/Precalculus — Name
Properties of Logs Investigation — Date
Day ______ — Block

**1)** In the days before calculators, people used tables to find logarithms. Natalie recently discovered such a table while rummaging around in her attic. (We've reproduced it for you to the right.) Each entry is accurate to 10 decimal places.

| | |
|---|---|
| log 1 | = 0 |
| log 2 | = 0.3010299957 |
| log 3 | = 0.4771212547 |
| log 4 | = 0.6020599913 |
| log 5 | = 0.6989700043 |
| log 6 | = 0.7781512504 |
| log 7 | = 0.84509804 |
| log 8 | = 0.903089987 |
| log 9 | = 0.9542425094 |
| log 10 | = 1 |
| log 11 | = 1.0413926852 |
| log 12 | = 1.0791812460 |

a) Natalie was curious about the table, so she checked a couple of entries on her calculator. You can do this, too. Are they correct?

b) Having some time to kill, Natalie decided to look for patterns. She thought $\log 2 + \log 3$ might be equal to log 5. Is it?

c) After computing $\log 2 + \log 3$ on her calculator, Natalie stared at her table again and discovered something interesting. What was it?

d) Natalie thought this was cool, so she kept looking. She next tried $\log 2 + \log 4$. What did she discover?

e) Complete and check: $\log 3 + \log 4 = ?$

f) Natalie thought the pattern broke down because $\log 5 + \log 2 = 1$. What do you think?

g) Copy and complete: $\log x + \log y =$ _________.

**2)** When Natalie told Sele about her discovery, Sele wondered if it was still true when the base wasn't 10. So she decided to check with some simple cases. Check whether or not $\log_b x + \log_b y = \log_b xy$ if:

a) $b = 3, x = 9, y = 27$

b) $b = 4, x = 8, y = 4$

c) $b = 5, x = \frac{1}{5}, y = \frac{1}{25}$

**3)** Natalie was so excited about her discovery that she told her friend Julio about it. Julio was impressed, and thought he could take it a step farther.

| | |
|---|---|
| log 1 | = 0 |
| log 2 | = 0.3010299957 |
| log 3 | = 0.4771212547 |
| log 4 | = 0.6020599913 |
| log 5 | = 0.6989700043 |
| log 6 | = 0.7781512504 |
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| log 12 | = 1.0791812460 |

a) He began by looking at $\log 6 - \log 2$. What did he discover?

b) Next, Julio tried $\log 8 - \log 4$. What did he discover?

c) Copy and complete: $\log x - \log y =$ ________.

4) Give a specific example to show that the following statement is **FALSE.**

$$\log\left(\frac{M}{N}\right) = \frac{\log M}{\log N}$$

5) Not to be outdone, Tarik decided to examine the table. Instead of adding two different entries, he decided to add the same entry to itself — in other words, to take multiples of one particular entry. He began with multiples of log 2. If the answer wasn't in the table, he used his calculator.

$\log 1 = 0$
$\log 2 = 0.3010299957$
$\log 3 = 0.4771212547$
$\log 4 = 0.6020599913$
$\log 5 = 0.6989700043$
$\log 6 = 0.7781512504$
$\log 7 = 0.84509804$
$\log 8 = 0.903089987$
$\log 9 = 0.9542425094$
$\log 10 = 1$
$\log 11 = 1.0413926852$
$\log 12 = 1.0791812460$

a) $2 \log 2 = ?$

b) $3 \log 2 = ?$

c) $4 \log 2 = ?$

d) Complete and check: $-2 \log 5 = ?$

e) Complete and check: $0.5 \log 64 = ?$

f) Copy and complete: $n \log x =$________ .

6) **Add these Laws of Logarithms to your notes:**

**Laws of Logarithms**

1. $\log(xy) = \log x + \log y$ — **Product Rule**
2. $\log\left(\frac{x}{y}\right) = \log x - \log y$ — **Quotient Rule**
3. $\log(x^n) = n \cdot \log x$ — **Power Rule**

7) Use a log law to write each of these in a different equivalent form, or say "impossible" and explain why.

a) $\log_5 8 + \log_5 4$ b) $\log(5 \cdot 7)$ c) $(\log_5 8)(\log_5 7)$

d) $\log(5 + 7)$ e) $\log_5 8 + \log_8 5$ f) $\log_3 5 + \log_3 5$

Trigonometry/Precalculus | Name
3.3 Properties of Logs | Date
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1) Use Properties of Logs to find the value of each of the following:

Let $\log_b M = 3.2$, $\log_b N = -1.5$, and $\log_b P = 2.4$. Find:

a) $\log_b \frac{M^2N}{P}$

b) $\log_b \frac{1}{M}$

c) $\log_b \sqrt[3]{\frac{P}{N}}$

d) $\log_b M^2\sqrt{P}$

2) Rewrite using the Change of Base Formula. Then find each value to the nearest .001.

a) $\log_{15} 1460$

b) $\log_{1/2} 4$

3) Rewrite each logarithm as a sum or quotient to approximate the following using the properties of logs, given the following values. Round your answer to 3 decimal places.

a) $\log_b \sqrt{3}$

b) $\log_b 30$

c) $\log_b \frac{16}{25}$

$log_b 2 = .356$

$log_b 3 = .565$

$log_b 5 = .827$

**4)** Find the value of each logarithm without using a calculator.

a) $\log_3 9$ b) $\log_6 \sqrt[3]{6}$ c) $\log_4 16^{3.4}$

d) $\log_5 0.04$ e) $\log_{100} 0.01$ f) $\log_9 \frac{1}{3}$

**5)** Solve each equation.

a) $\log_4 (x + 3) = -1$ b) $\log (x^2 - 4) = 0$

c) $\log 3x^2 = \log (2x + 15)$ d) $log_3\ 8x - 3 = log_3\ 2x + 9$

**6) Simplify each of the following using Properties of Logs.**

**a)** $\ln y + \ln z$ **b)** $\log_5 8 - \log_5 t$

**c)** $2 \log_2(x + 3)$ **d)** $\ln x - 3 \ln(x + 1)$

**e)** $3 \ln x + 2 \ln y - 4 \ln z$ **f)** $\ln x - 2[\ln(x + 2) + \ln(x - 2)]$

**7) Find the exact value of the following without using a calculator.**

**a)** $\log_5 75 - \log_5 3$ **b)** $\log_4 2 + \log_4 32$ **c)** $\ln e^6 - 2 \ln e^5$