5.1 Trig Identities

= equivalent expressions used to simplify more complex expressions

Some we've used already:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

New identities:

$\tan\theta = \frac{Y}{X}$ ← $\sin\theta$, X ← $\cos\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

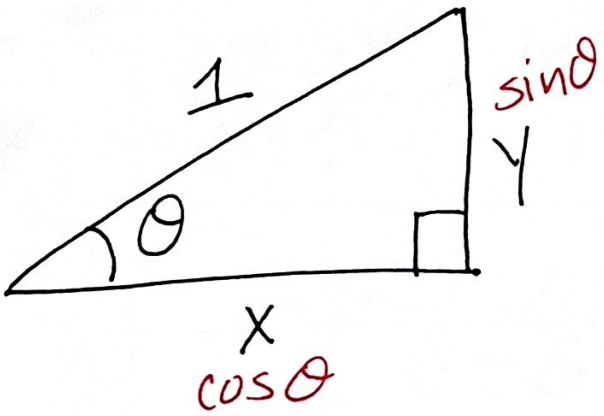
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

ex: Simplify $(\cot\theta)(\sin\theta)$

$$\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right)\left(\frac{\cancel{\sin\theta}}{1}\right)$$

$$\boxed{\cos\theta}$$

Pythagorean Identities



$x^2 + y^2 = 1^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1^2$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Strategies for simplifying/verifying trig identities:

1) Replace expressions w/sine or cosine equivalents.
2) Find common denominators.
3) Factor.
4) Work on most complicated side first.

ex: $\frac{\csc^2\theta}{\cot\theta} - \frac{\cot\theta}{1}\left(\frac{\cot\theta}{\cot\theta}\right)$

* Find common denominators

$\frac{\csc^2\theta - \cot^2\theta}{\cot\theta}$ ooo

$\frac{1}{\cot\theta}$

$\boxed{\tan\theta}$

$1 + \cot^2\theta = \csc^2\theta$

$-\cot^2\theta \quad -\cot^2\theta$

$1 = \csc^2\theta - \cot^2\theta$

ex: Show that

$\left(\frac{1+\sin x}{1+\sin x}\right)\frac{1}{1-\sin x} + \frac{1}{1+\sin x}\left(\frac{1-\sin x}{1-\sin x}\right) = 2\sec^2 x$

$\frac{1+\sin x + 1-\sin x}{(1-\sin x)(1+\sin x)} =$

$\frac{2}{1-\sin^2 x} =$

$\frac{2}{\cos^2 x} =$

$2\left(\frac{1}{\cos^2 x}\right) =$

$2\sec^2 x = \checkmark$