

(5.4 continued)

Name Alex Carr

Sum & Difference Identities for Tangent

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

ex: Find the exact value of $\tan 105^\circ \leftarrow 60^\circ + 45^\circ$
 $\alpha \quad \beta$

$$\tan(60^\circ + 45^\circ) = \frac{\tan 60^\circ + \tan 45^\circ}{1 - (\tan 60^\circ)(\tan 45^\circ)}$$

$$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$$

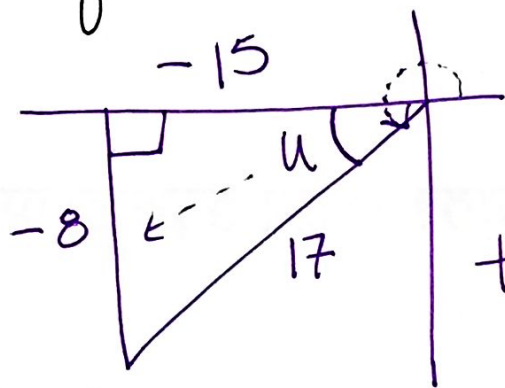
$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})}$$

$$= \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3}$$

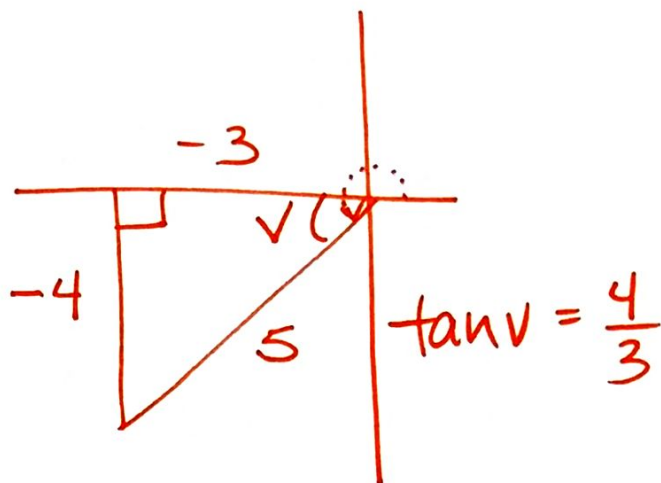
$$= \frac{4 + 2\sqrt{3}}{-2}$$

$$= \boxed{-2 - \sqrt{3}}$$

ex: Find $\tan(u+v)$ given $\sin u = \frac{-8}{17}$ and $\cos v = -\frac{3}{5}$ where u & v both lie in quadrant III.



$$\tan u = \frac{8}{15}$$



$$\tan v = \frac{4}{3}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\frac{\frac{8}{15} + \frac{4}{3} \left(\frac{5}{5}\right)}{1 - \left(\frac{8}{15}\right) \left(\frac{4}{3}\right)} = \frac{\frac{8}{15} + \frac{20}{15}}{\frac{45}{45} - \frac{32}{45}}$$

$$= \frac{28}{15} \div \frac{13}{45} = \frac{28}{15} \cdot \frac{45}{13} = \frac{84}{13}$$

ex: Rewrite as the sin/cos/tan of one angle using sum & diff. identities.

$$a) \cos 21^\circ \cos 16^\circ - \sin 21^\circ \sin 16^\circ = \cos(21 + 16) = \cos 37^\circ$$

$$b) \frac{\tan 41^\circ - \tan 15^\circ}{1 + \tan 41^\circ \tan 15^\circ} = \tan(41 - 15) = \tan 26^\circ$$