Double and Half Angle Identities

-additional tools to use in order to find exact values of trigonometric functions, or to simply expressions involving trigonometric functions.

We can use sum and difference identities to derive all 3 Double Angle Identities.

Expand and simplify each of the following:

$\sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$

$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\boxed{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha}$

$1-\sin^2\alpha$ $\quad$ $1-\cos^2\alpha$

$\boxed{\cos 2\alpha = 1 - 2\sin^2\alpha}$ $\quad$ $\boxed{\cos 2\alpha = 2\cos^2\alpha - 1}$

$\tan(\alpha+\alpha) = \dfrac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$

$\boxed{\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}}$

Half Angle Identities

$\sin\dfrac{u}{2} =$ $\qquad$ $\cos\dfrac{u}{2} =$

**Signs of sine and cosine depend on the quadrant in which $\frac{u}{2}$ lies.*

$\tan\dfrac{u}{2} =$ $\qquad$ or $\qquad$ $\tan\dfrac{u}{2} =$

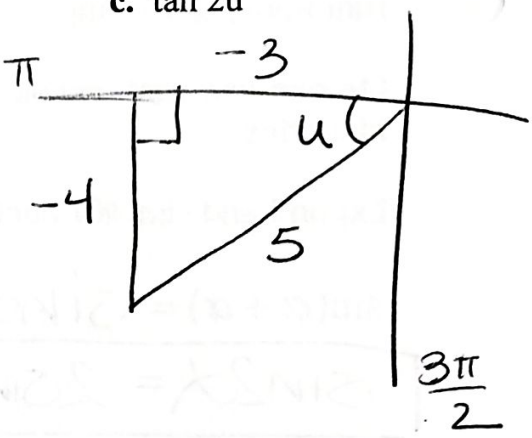
Ex 1:

Given $\cos u = -\frac{3}{5}$ with $\pi < u < \frac{3\pi}{2}$, find the following.

$\sin u = -\frac{4}{5}$

$\tan u = \frac{4}{3}$

a. $\sin 2u$ **b.** $\sin \frac{u}{2}$ **c.** $\tan 2u$



a) $\sin 2u = 2\sin u \cos u$

$= 2\left(-\frac{4}{5}\right)\left(\frac{-3}{5}\right)$

$= \frac{24}{25}$

c) $\tan 2u = \frac{2\tan u}{1-\tan^2 u}$

$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{\frac{9}{9}-\frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} = \frac{8}{3} \div \frac{-7}{9} = \frac{8}{3} \cdot \frac{3}{-7}$

$= \frac{-24}{7}$

Ex 2: Use a half angle identity to find the exact value of cos 105°.

Ex 3: Use double angle identities to solve each of the following equations

a) $\cos 2x + \cos x = 0$ for all values of x.

$u = \cos x$

$2\cos^2 x - 1 + \cos x = 0$

$2u^2 + u - 1 = 0$

$(2u - 1)(u + 1) = 0$

$2u - 1 = 0 \quad u = -1$

$u = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n$

$\cos x = -1$

$x = \pi + 2\pi n$

b) $\tan 2x + \tan x = 0$ for $0 \le x < 2\pi$.

$\frac{(1-\tan^2 x)}{1} \cdot \frac{2\tan x}{1-\tan^2 x} + \tan x = 0 \, (1-\tan^2 x)$

$2\tan x + \tan x(1-\tan^2 x) = 0$

$2\tan x + \tan x - \tan^3 x = 0$

$3\tan x - \tan^3 x = 0$

$\frac{}{\tan x} \quad \frac{}{\tan x}$

$\tan x(3 - \tan^2 x) = 0$

$\tan x = 0 \quad 3 - \tan^2 x = 0$

$x = 0$

$\sqrt{\tan^2 x} = \sqrt{3}$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$