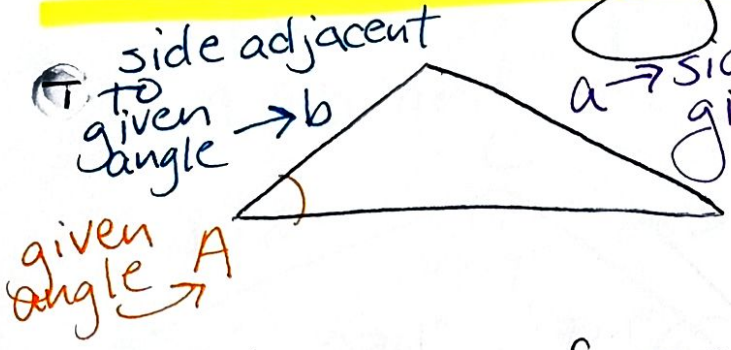
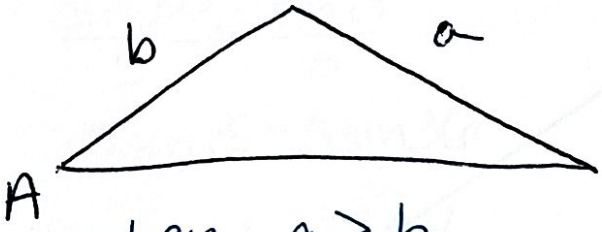
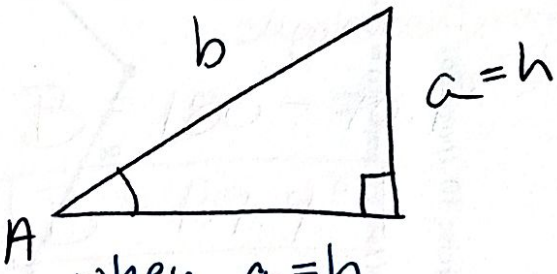
6.1 (cont.) Ambiguous Case of Law of Sines



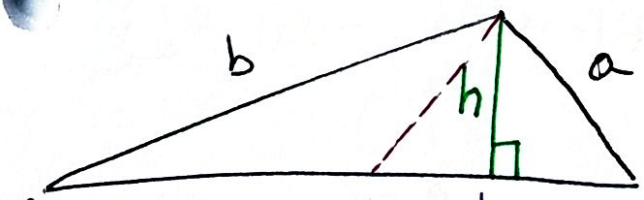
In the case of SSA...



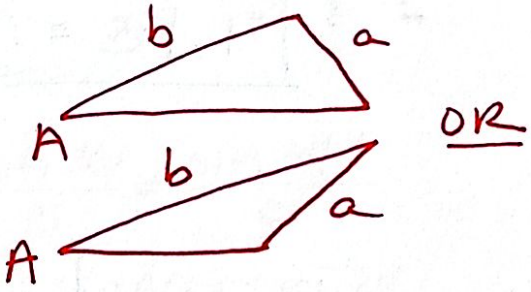
when $a > b$
→ one possible △



when $a = h$
→ one possible △



when $a < b$
→ 2 △'s possible

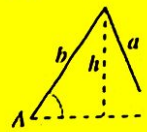
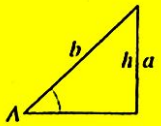
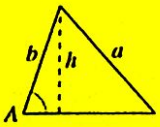
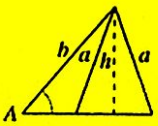
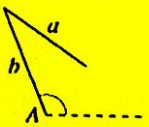
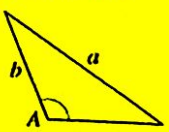


$\sin A = \frac{h}{b}$ $h = b\sin A$

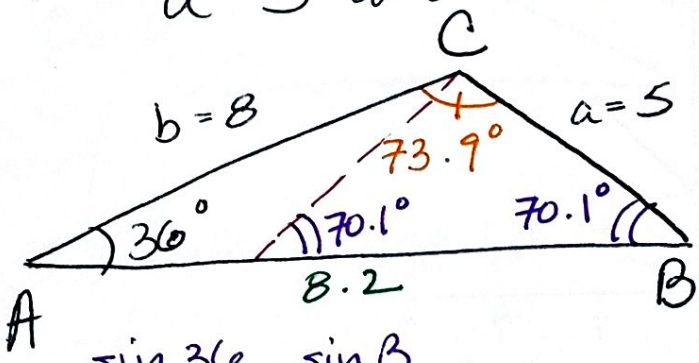
Test ⇒ $b\sin A < a < b$ ← if this is true, 2 △'s can be drawn.

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. (Notice that $h = b \sin A$.)

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Possible triangles	None	One	One	Two	None	One

ex: Find the missing angles & sides of the 2 Δ's possible when $A = 36°$, $a = 5$ and $b = 8$.



$\frac{\sin 36}{5} = \frac{\sin B}{8}$

$\frac{5 \sin B}{5} = \frac{8 \sin 36}{5}$

$\sin^{-1} \sin B = \sin^{-1} .94$

$\boxed{B = 70.1°}$

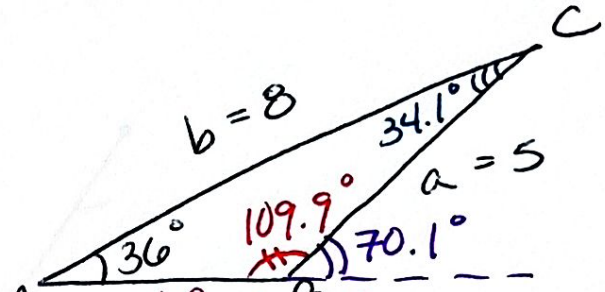
$C = 180 - (36 + 70.1)$

$\boxed{C = 73.9°}$

$\frac{\sin 36}{5} = \frac{\sin 73.9}{c}$

$\frac{c \sin 36}{\sin 36} = \frac{5 \sin 73.9}{\sin 36}$

$\boxed{c = 8.2}$



supplementary ∠'s

$B = 180 - 70.1°$

$\boxed{B = 109.9°}$

$C = 180 - (109.9 + 36)$

$\boxed{C = 34.1°}$

$\frac{\sin 36}{5} = \frac{\sin 34.1}{c}$

$\frac{c \sin 36}{\sin 36} = \frac{5 \sin 34.1}{\sin 36}$

$\boxed{c = 4.8}$

ex: Determine whether a Δ is possible given $A = 60°$, $a = 4$ and $b = 14$.

$\frac{\sin 60}{4} = \frac{\sin B}{14}$

$\frac{4 \sin B}{4} = \frac{14 \sin 60}{4}$

$\sin^{-1} \sin B = \sin^{-1} 3.03$

since can't sine be greater than 1, this Δ is not possible.