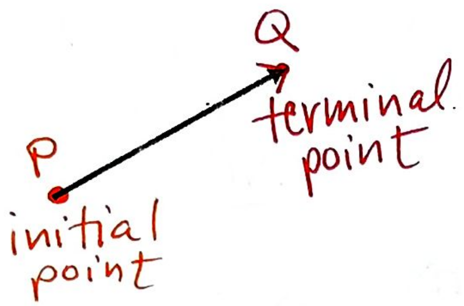
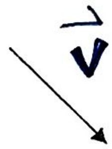
6.3 Notes: *Introduction to Vectors in the Plane*

Vector = a directed line segment that represents force ***and*** direction.

$\overrightarrow{PQ}$





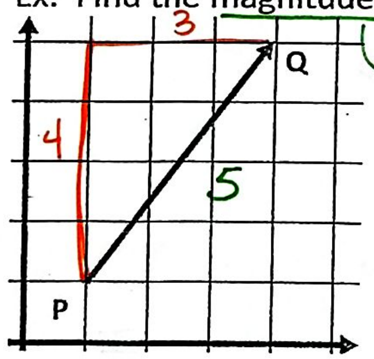
Magnitude = size or amount of force

The length **of the vector represents its magnitude.**

The magnitude of any vector can be found by using the Pythagorean Theorem or distance formula.

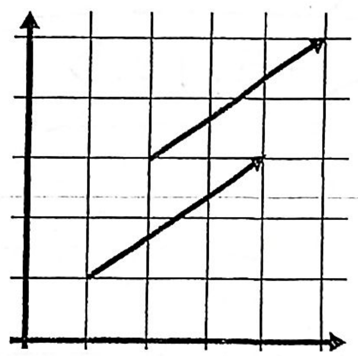
↳ $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Ex: Find the magnitude of $\overrightarrow{PQ}$.



→ $\|\overrightarrow{PQ}\| = 5$

Two vectors are equivalent **when they have the same magnitude and direction.**



Component Form of a Vector

The directed line segment whose initial point is the origin is often the most convenient. This vector **v** is in component form.

The component form of a vector **v**, is written as:

$$\vec{v} = \langle x, y \rangle$$

The component form of a vector with initial point $P(\cancel{p_1, p_2})$ (x_1, y_1) and terminal point $Q(\cancel{q_1, q_2})$ (x_2, y_2) is given by

terminal − initial

$$V = \langle x_2 - x_1, y_2 - y_1 \rangle$$

The magnitude (or length) of **v** is given by

$$\boxed{\|v\| = \sqrt{x^2 + y^2}}$$

Ex: Show that vectors **u** and **v** are equal:

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The vector **u** from P(0,0) to Q(3,2).
The vector **v** from R(1,2) to S(4,4)

$u = \langle 3-0, 2-0 \rangle$ $V = \langle 4-1, 4-2 \rangle$

$u = \langle 3, 2 \rangle$ $V = \langle 3, 2 \rangle$

same component form = equivalent vectors

Ex: Find the component form and magnitude of the vector v that has initial point $(4, -7)$ and terminal point $(-1, 5)$

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$V = \langle -1-4, 5+(+7) \rangle$

$$\boxed{V = \langle -5, 12 \rangle}$$

$\|v\| = \sqrt{(-5)^2 + 12^2}$

$= \sqrt{169}$

$$\boxed{\|v\| = 13}$$

YOU TRY!

Find the component form and magnitude of the vector v that has initial point $(2, 2)$ and terminal point $(-1, 4)$

$\|v\| = \sqrt{(-3)^2 + 2^2}$

$$\boxed{V = \langle -3, 2 \rangle}$$

$$\boxed{\|v\| = \sqrt{13}}$$

Vector Operations

Two basic vector operations are scalar multiplication and vector addition.

Illustration of Scalar Multiplication

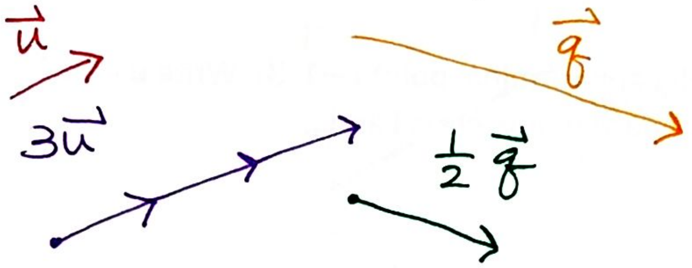
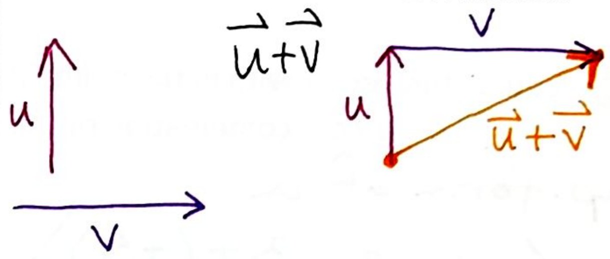


Illustration of Vector Addition



Let $\boldsymbol{u} = \langle \cancel{u_1, u_2} \rangle$ and $\boldsymbol{v} = \langle \cancel{v_1, v_2} \rangle$ be vectors and let k be a scalar (a real number), then:
$\langle x_1, y_1 \rangle$ $\langle x_2, y_2 \rangle$

sum of **u** and **v** is the vector	scalar multiple of k times **u** is the vector
$\vec{u}+\vec{v} = \langle x_1 + x_2, y_1 + y_2 \rangle$	$k\langle x_1, y_1 \rangle \Rightarrow k\vec{u} = \langle kx_1, ky_1 \rangle$
negative of $\boldsymbol{v} = \langle v_1, v_2 \rangle$	difference of **u** and **v**
$-\vec{v} = \langle -x_2, -y_2 \rangle$	$\vec{u}-\vec{v} = \langle x_1 - x_2, y_1 - y_2 \rangle$

Let $\boldsymbol{v} = \langle -2, 5 \rangle$ *and* $\boldsymbol{w} = \langle 3, 4 \rangle$, find the each of the following vectors

a. **2v**	$2\vec{v} = 2\langle -2, 5 \rangle$ $= \langle -4, 10 \rangle$
b. $\boldsymbol{w} - \boldsymbol{v}$	$\vec{w}-\vec{v} = \langle 3-(-2), 4-5 \rangle$ $= \langle 5, -1 \rangle$
c. $\boldsymbol{v} + 2\boldsymbol{w}$	$v + 2w = \langle -2, 5 \rangle + 2\langle 3, 4 \rangle$ $= \langle -2, 5 \rangle + \langle 6, 8 \rangle$ $= \langle 4, 13 \rangle$

The letters **i** and **j** can also be used to represent a vector's components. This is called a linear combination.

$xi + yj$

Ex: let **u** be the vector with initial point $\overset{I}{(2,-5)}$ and terminal point $\overset{T}{(-1,3)}$. Write **u** as a linear combination of the standard unit vectors **i** and **j**.

comp. form of u

$u = \langle -1-2,\ 3+(+5) \rangle$

$u = \langle -3, 8 \rangle$

$\Rightarrow -3i + 8j$

Ex: Let $\boldsymbol{u} = -3\boldsymbol{i} + 8\boldsymbol{j}$ *and* $\boldsymbol{v} = 2\boldsymbol{i} - \boldsymbol{j}$. Find $2\boldsymbol{u} - 3\boldsymbol{v}$

$2(-3i+8j) + {}^{-}3(2i-j)$

$-6i + 16j - 6i + 3j$

$\boxed{-12i + 19j}$

Finding Direction Angles of Vectors

Direction angles of vectors are measured in several different ways. Today we will measure our angles in standard form, counter-clockwise from the positive x-axis.

Find the direction angle of each of the following vectors.

Ex: $v\langle 4,8 \rangle$

Ex: $v\langle -6,-2 \rangle$