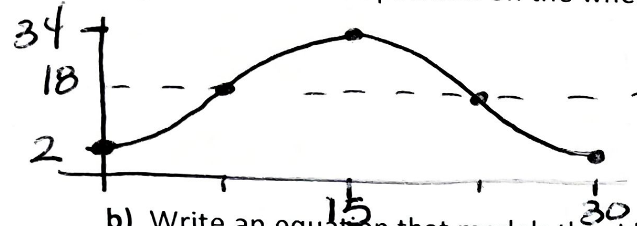
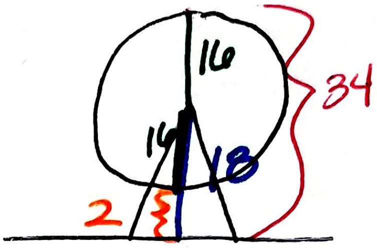
Trigonometry/Precalculus
Chapter 5 Review
Day ______

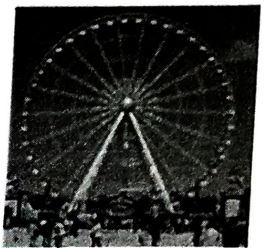
Name
Date
Block

1) At a county fair, the Ferris wheel has a diameter of 32m and its center is 18 m above the ground. The wheel completes one revolution every 30 seconds.

a) Graph a rider's height above the ground, in meters, versus the time in seconds. Assume the rider begins at the lowest position on the wheel.







b) Write an equation that models the rider's height with respect to time.

$p = 30$

$\frac{30b}{30} = \frac{2\pi}{30}$ $\quad b = \frac{\pi}{15}$

$y = -16\cos\left(\frac{\pi}{15}x\right) + 18$

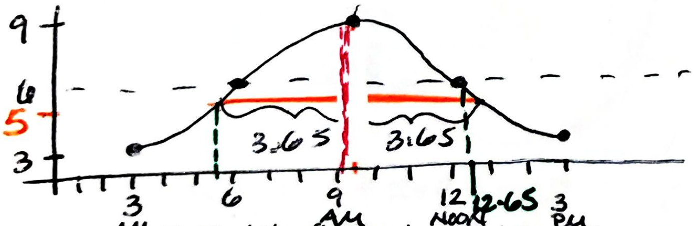
$y = 16\cos\left(\frac{\pi}{15}(x-15)\right) + 18$

$y = 16\sin\left(\frac{\pi}{15}(x-7.5)\right) + 18$

2) At the high tide the water level at a particular boat dock is 9 feet deep. At low tide the water is 3 feet deep. On a certain day the low tide occurs at 3 AM and high tide occurs at 9AM.
The equation to model the tide height (y) with respect to hours after midnight (x) is given below.

$$y = 3\cos\left(\frac{\pi}{6}(x-9)\right) + 6$$

a) Using this data, sketch a sinusoidal curve to model the tides height throughout the day.



b) What is the water level at 2 PM? ← 14 hours past midnight

$y = 3\cos\left(\frac{\pi}{6}(14-9)\right) + 6$

$= 3\cos\left(\frac{\pi}{6}(5)\right) + 6$

$= 3\cos\frac{5\pi}{6} + 6$

$= 3\left(-\frac{\sqrt{3}}{2}\right) + 6$

$y = -\frac{3\sqrt{3}}{2} + 6$

$\boxed{y = 3.4 \text{ feet}}$

c) Find the first and second time after midnight that the water reaches a level of 5 feet.

$5 = 3\cos\left(\frac{\pi}{6}(x-9)\right) + 6$
$-6 \qquad -6$

$\frac{-1}{3} = \frac{3\cos\left(\frac{\pi}{6}(x-9)\right)}{3}$

$\cos^{-1} -\frac{1}{3} = \cos^{-1}\cos\left(\frac{\pi}{6}(x-9)\right)$

$\frac{6}{\pi}(1.911) = \left(\frac{\pi}{6}(x-9)\right)\frac{6}{\pi}$

$3.65 = x - 9$
$+9 \qquad +9$

$x = 12.65$

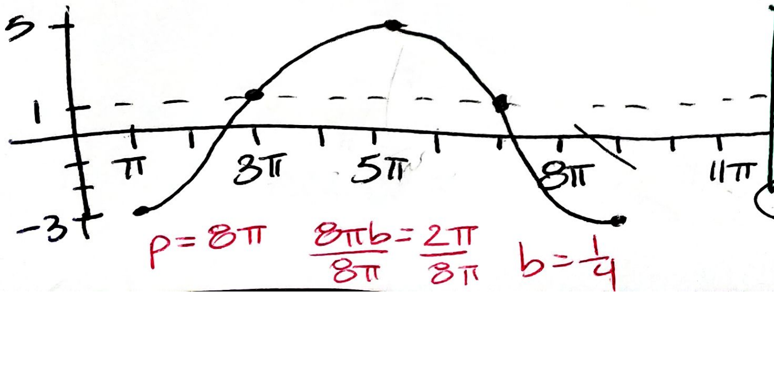
$12.65 - 9 = 3.65$

$9 - 3.65 = 5.35$

$5.35 \times 60 = $ 5:21 AM

$12.65 \times 60 = $ 12:39 PM

3) Write an equation for the sinusoidal function that passes through $(\pi, -3)$ and $(5\pi, 5)$.



$y = 4\sin\frac{1}{4}(x - 3\pi) + 1$

$y = 4\cos\left(\frac{1}{4}(x - 5\pi)\right) + 1$

$p = 8\pi \qquad \frac{8\pi b}{8\pi} = \frac{2\pi}{8\pi} \qquad b = \frac{1}{4}$

4) Simplify each of the following using trigonometric identities.

a) $\sec x - \tan x \sin x$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \sin x$$

$$\frac{1-\sin^2 x}{\cos x}$$

$$\frac{\cos^{\cancel{2}} x}{\cancel{\cos x}}$$

$$\boxed{\cos x}$$

b) $\dfrac{1}{\sin x \cot x} =$

$$\frac{1}{\cancel{\sin x}\left(\frac{\cos x}{\cancel{\sin x}}\right)}$$

$$\frac{1}{\cos x}$$

$$\boxed{\sec x}$$

c) $(1+\tan^2\theta)\cos^2\theta$

$$\sec^2\theta \cdot \cos^2\theta$$

$$\frac{1}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta}$$

$$\boxed{1}$$

5) Verify each of the following identities:

a) $\sin x\,(\cot x + \tan x) = \sec x$

$$\sin x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$$

$$\frac{\cancel{\sin x}\cdot \cos x}{\cancel{\sin x}} + \frac{\sin^2 x}{\cos x} =$$

$$\frac{\cos x}{\cos x}(\cos x) + \frac{\sin^2 x}{\cos x} =$$

$$\frac{\underbrace{\cos^2 x + \sin^2 x}_{1}}{\cos x} =$$

$$\frac{1}{\cos x} =$$

$$\sec x = \checkmark$$

b) $\sin(x+y) - \sin(x-y) = 2\cos x \sin y$

$$\sin x\cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y)$$

$$\sin x \cancel{\cos y} + \cos x \sin y - \sin \cancel{x} \cos y + \cos x \sin y$$

$$2\cos x \sin y = \checkmark$$

c) $\csc\theta\cos^2\theta + \sin\theta = \csc\theta$

$$\left(\frac{1}{\sin\theta}\right)\cos^2\theta + \sin\theta =$$

$$\frac{\cos^2\theta}{\sin\theta} + \sin\theta\,\frac{\sin\theta}{\sin\theta} =$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta} =$$

$$\frac{1}{\sin\theta} =$$

$$\csc\theta = \checkmark$$

d) $\dfrac{(1+\sin\alpha)}{(1+\sin\alpha)}\dfrac{1}{1-\sin\alpha} + \dfrac{1}{1+\sin\alpha}\dfrac{(1-\sin\alpha)}{(1-\sin\alpha)} = 2\sec^2\alpha$

$$\frac{1+\cancel{\sin\alpha} + 1 - \cancel{\sin\alpha}}{1-\sin^2\alpha} =$$

$$\frac{2}{\cos^2\alpha} =$$

$$2\left(\frac{1}{\cos^2\alpha}\right) =$$

$$2\sec^2\alpha = \checkmark$$

6) Solve each of the following equation for the given intervals.

a) $(\sec x+2)(\sqrt{2}\sin x-1)=0$ for all values of x

$\sec x+2=0$ $\quad \sqrt{2}\sin x-1=0$

$\sec x=-2$ $\quad +1 \quad +1$

$\frac{1}{\cos x}=-2$ $\quad \frac{\sqrt{2}\sin x}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\cos x=-\frac{1}{2}$ $\quad \sin x=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\boxed{x=\frac{2\pi}{3},\frac{4\pi}{3}+2\pi n}$ $\quad \sin x=\frac{\sqrt{2}}{2}$

$\boxed{x=\frac{\pi}{4},\frac{3\pi}{4}+2\pi n}$

b) $\tan^2 x-3=0$ for $[0, 2\pi)$

$\sqrt{\tan^2 x}=\sqrt{3}$

$\tan x=\pm\sqrt{3}$

$\boxed{x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}}$

c) $2\cos^2 x-\cos x-1=0$ for all values of x

$u=\cos x$

$2u^2-u-1=0$

$(2u+1)(u-1)=0$

$2u+1=0$ $\quad u-1=0$

$u=-\frac{1}{2}$ $\quad u=1$

$\cos x=-\frac{1}{2}$ $\quad \cos x=1$

$\boxed{x=\frac{2\pi}{3},\frac{4\pi}{3}+2\pi n}$ $\boxed{x=0+2\pi n}$

d) $\cos^2 x-\sin x-1=0$ for $[0, 2\pi)$

$1-\sin^2 x-\sin x-1=0$

$-1\,(-\sin^2 x-\sin x)=(0)\,-1$

$\sin^2 x+\sin x=0$

$\sin x(\sin x+1)=0$

$\sin x=0$ $\quad \sin x+1=0$

$\boxed{x=0,\pi}$ $\quad \sin x=-1$

$\boxed{x=\frac{3\pi}{2}}$

e) $\sin 3x-1=0$ for all values of x

$u=3x$

$\sin u-1=0$

$\sin u=1$

$u=\frac{\pi}{2}+2\pi n$

$\frac{1}{3}(3x)=\left(\frac{\pi}{2}+2\pi n\right)\frac{1}{3}$

$\boxed{x=\pi/6+\frac{2\pi n}{3}}$

f) $\cos 4x(\csc x+\sqrt{2})=0$ for $[0, 2\pi)$

$u=4x$ $\quad \cos 4x=0$ $\quad \csc x+\sqrt{2}=0$

$\cos u=0$ $\quad \csc x=-\sqrt{2}$

$u=\frac{\pi}{2},\frac{3\pi}{2}$ $\quad \frac{1}{\sin x}=\frac{-\sqrt{2}}{1}$

$\frac{1}{4}(4x)=\left(\frac{\pi}{2}\right)\frac{1}{4}$ $\quad 4x=\left(\frac{3\pi}{2}\right)\frac{1}{4}$ $\quad \sin x=-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\boxed{x=\frac{\pi}{8} \quad x=\frac{3\pi}{8}}$ $\quad \sin x=-\frac{\sqrt{2}}{2}$

$\boxed{x=\frac{5\pi}{4},\frac{7\pi}{4}}$

Use sum and difference identities to find the exact value of the following:

7) $\sin 105^0$

$\sin(45+60)=$

$=\sin 45\cos 60+\cos 45\sin 60$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

8) $\cos 15^0$

$\cos(60-45)$

$=\cos 60\cos 45+\sin 60\sin 45$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

9) $\tan 255^0$

$\tan(135+120)$

$=\frac{\tan 135+\tan 120}{1-\tan 135\tan 120}$

$=\frac{-1+(-\sqrt{3})}{1-(+1)(+\sqrt{3})}$

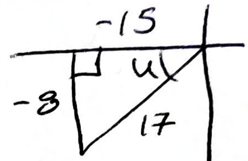
$\frac{-1-\sqrt{3}(1+\sqrt{3})}{1-\sqrt{3}(1+\sqrt{3})}$

$\frac{-1-\sqrt{3}-\sqrt{3}-3}{1+\sqrt{3}-\sqrt{3}-3}$

$=\frac{-4-2\sqrt{3}}{-2}$

$\boxed{2+\sqrt{3}}$

If $\cos u = -\frac{15}{17}$ and $\pi < u < \frac{3\pi}{2}$, find the exact value for each of the following:



$\sin u = \frac{-8}{17}$

$\tan u = \frac{8}{15}$

10) sin u

$\boxed{\frac{-8}{17}}$

11) sin 2u

$2\sin u\cos u$

$2\left(\frac{-8}{17}\right)\left(\frac{-15}{17}\right) = \boxed{\frac{240}{289}}$

12) cos 2u

$\cos^2 x - \sin^2 x$

$\left(\frac{-15}{17}\right)^2 - \left(\frac{-8}{17}\right)^2$

$\frac{225}{289} - \frac{64}{289} = \boxed{\frac{161}{289}}$

13) tan 2u

$\frac{2\tan u}{1-\tan^2 u} = \frac{2\left(\frac{8}{15}\right)}{1-\left(\frac{8}{15}\right)^2} = \frac{\left(\frac{16}{\cancel{15}}\right)\ \cancel{225}\ 15}{\left(1-\frac{64}{225}\right)\ 225}$

$\frac{240}{225-64} = \boxed{\frac{240}{161}}$

14) $\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}}$

$= -\sqrt{\frac{1+\left(\frac{-15}{17}\right)\ 17}{2\ \ 17}} = -\sqrt{\frac{17-15}{34}} = -\sqrt{\frac{2}{34}}$

$= -\sqrt{\frac{1}{17}} = -\frac{1}{\sqrt{17}}\left(\frac{\sqrt{17}}{\sqrt{17}}\right) = \boxed{\frac{-\sqrt{17}}{17}}$

15) $\tan\frac{u}{2}$

$= \frac{\sin u}{1+\cos u}$

$= \frac{\left(\frac{-8}{17}\right)\ 17}{\left(1+\left(\frac{-15}{17}\right)\right)17} = \frac{-8}{17-15} = \frac{-8}{2} = \boxed{-4}$

Use trig identities to write each expression as the sine, cosine or tangent of one angle.

16) $\cos 11^\circ \cos 18^\circ - \sin 11^\circ \cos 18^\circ$

$\alpha \quad \beta \quad \alpha \quad \beta$

sin

$\cos(11+18)$

$\boxed{\cos 29^\circ}$

17) $\sin 50^\circ \cos 23^\circ - \sin 50^\circ \cos 23^\circ$

cos sin

$\sin(50-23)$

$\boxed{\sin 27^\circ}$

18) $\frac{\tan 34^\circ + \tan 10^\circ}{1 - \tan 34^\circ \tan 10^\circ}$

$\tan(34+10)$

$\boxed{\tan 44^\circ}$

Find the solutions to the following equations on the interval [0, 2π).

19)

$\cos\left(x+\frac{\pi}{6}\right) - \cos\left(x-\frac{\pi}{6}\right) = 1$

$\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6} - \left(\cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6}\right) = 1$

$\cancel{\cos x\frac{\sqrt{3}}{2}} - \sin x\frac{1}{2} - \cancel{\cos x\frac{\sqrt{3}}{2}} - \sin x\frac{1}{2} = 1$

$-\frac{1}{2}\sin x - \frac{1}{2}\sin x = 1$

$-1\sin x = 1$

$\sin x = -1$

$\boxed{x = \frac{3\pi}{2}}$

20)

$\sin 2x \sin x = \cos x$

$2\sin x\cos x\sin x = \cos x$

$2\sin^2 x\cos x - \cos x = 0$

$\cos x(2\sin^2 x - 1) = 0$

$\cos x = 0 \qquad 2\sin^2 x - 1 = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$2\sin^2 x = 1$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$