Trigonometry/Precalculus
Chapter 2 Review
Day __________ CW/HW

Name
Date
Block

1) Describe the end behavior of each of the following functions. (in $x \to$ form)

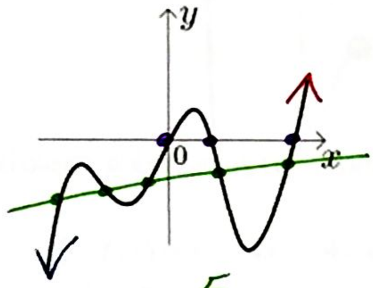
a) $f(x) = -2x^4 + 3x - 11$

As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to -\infty$

b) $g(x) = 10x^7 + 8x^6 - x^5 + 18x^2 + 1$

As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

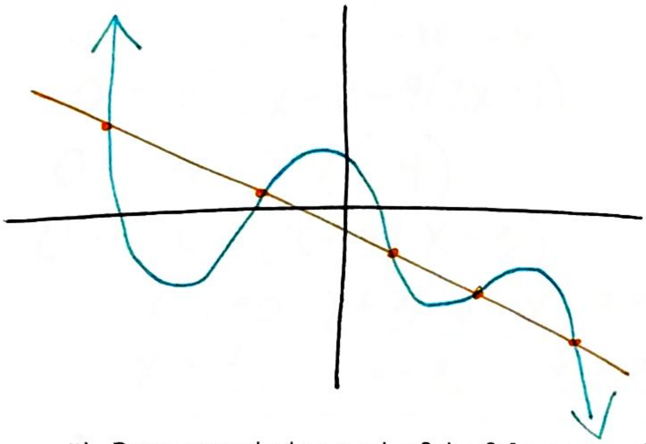
2) Identify the following for the function below:



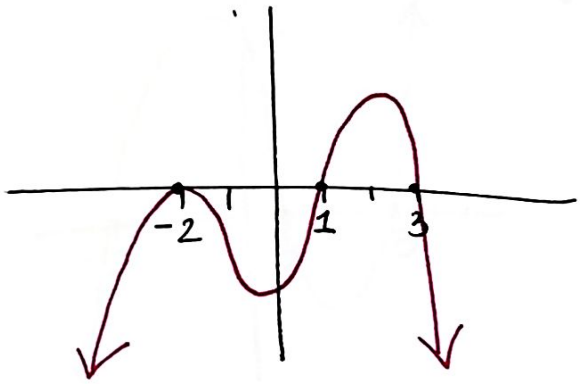
Degree = 5
Number of real zeros: 3
As $x \to \infty$, $f(x) \to$ ∞
As $x \to -\infty$, $f(x) \to$ $-\infty$

3) Sketch a polynomial function that meets the following specifications:

A) Degree 5,
With negative leading coefficient
3 real roots, 2 complex roots.



B) Degree 4,
as $x \to -\infty$, $f(x) \to -\infty$,
as $x \to \infty$, $f(x) \to -\infty$,
Double root at -2, single root at 1 and 3



4) Determine whether each of the following is a factor of each polynomial. **Show how you know.**

a) (x-2) a factor of $f(x) = x^4 + 5x^3 + 6x^2 - x - 2$?

2	1	5	6	-1	-2
	↓	2	14	40	78
	1	7	20	39	76

x-2 is not a factor because it does not divide evenly into f(x).

b) (2x-3) a factor of $f(x) = 2x^3 - 3x^2 - 50x + 75$?

$2x - 3 = 0$
$2x = 3$
$x = \frac{3}{2}$

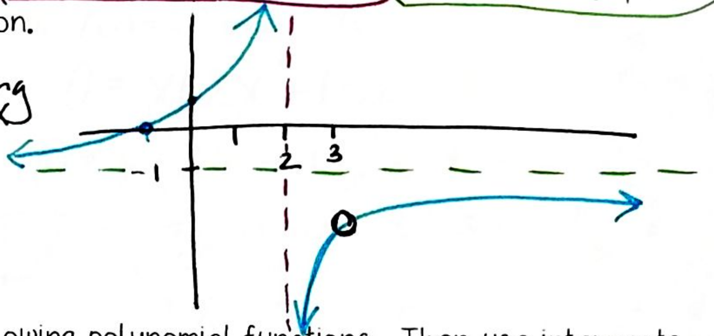
$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)^2 - 50\left(\frac{3}{2}\right) + 75$

$f\left(\frac{3}{2}\right) = 0$ ← since $f\left(\frac{3}{2}\right) = 0$, the factor theorem tells that 2x-3 is a factor.

5) Write a rational equation for a function with vertical asymptote at $x = 2$, horizontal asymptote at $x = -1$ and a hole at $x = 3$. Sketch the function.

$$f(x) = \frac{-(x-3)(x+1)}{(x-2)(x-3)}$$

← can be anything w/an x



6) Find the exact value of all zeros for the following polynomial functions. Then use intercepts and zeros to accurately sketch the function.

a) $f(x) = 3x^3 + 8x^2 - 33x + 10$ if 2 is a zero.

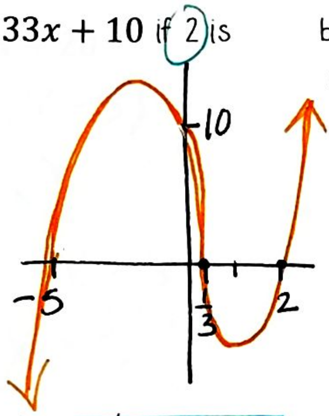
$$\begin{array}{r|rrrr} 2 & 3 & 8 & -33 & 10 \\ & \downarrow & 6 & 28 & -10 \\ \hline & 3 & 14 & -5 & 0 \end{array}$$

$0 = 3x^2 + 14x - 5$

$0 = (3x-1)(x+5)$

$3x-1=0 \quad x+5=0$

$\boxed{x = \frac{1}{3}, -5, 2}$



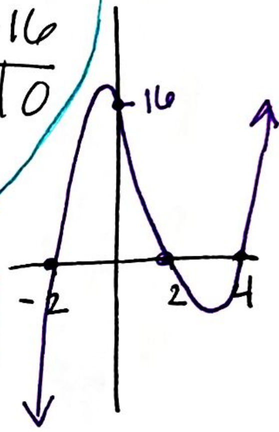
b) $f(x) = x^3 - 4x^2 - 4x + 16$ if $x - 4$ is a factor

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -4 & 16 \\ & \downarrow & 4 & 0 & -16 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$x^2 - 4 = 0$

$x^2 = 4$

$\boxed{x = \pm 2, 4}$



c) $f(x) = 2x^3 - x^2 - 18x + 9$

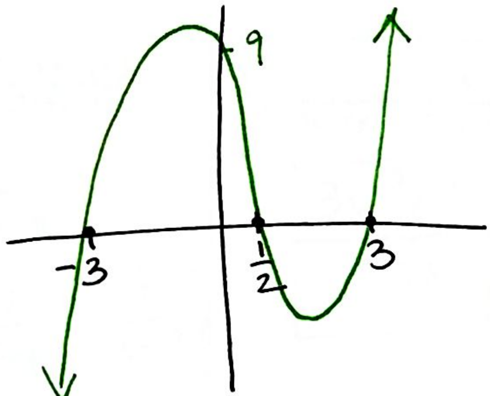
$0 = x^2(2x-1) - 9(2x-1)$

$0 = (2x-1)(x^2-9)$

$0 = (2x-1)(x+3)(x-3)$

$2x-1=0 \quad x+3=0 \quad x-3=0$

$\boxed{x = \frac{1}{2} \quad x = -3 \quad x = 3}$



7) Use your graphing calculator to find a rational root (zero) of the polynomial, then find the exact values of the remaining roots.

$f(x) = x^3 + 4x^2 - 16x - 15$ $\quad x = 3$ is a zero

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -16 & -15 \\ & \downarrow & 3 & 21 & 15 \\ \hline & 1 & 7 & 5 & 0 \end{array}$$

zeros at $x = 3, \frac{-7 \pm \sqrt{29}}{2}$

$0 = x^2 + 7x + 5$

$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(5)}}{2(1)} \Rightarrow x = \frac{-7 \pm \sqrt{29}}{2}$

8) Factor each of the following polynomials completely, then find the zeros of each function.

a) $f(x) = x^4 - 7x^2 + 12$

$0 = (x^2 - 3)(x^2 - 4)$

$x^2 - 3 = 0 \quad x^2 - 4 = 0$

$x^2 = 3 \quad x^2 = 4$

$\boxed{x = \pm\sqrt{3}} \quad \boxed{x = \pm 2}$

b) $f(x) = 2x^3 + 13x^2 - 7x$

$0 = x(2x^2 + 13x - 7)$

$0 = x(2x - 1)(x + 7)$

$\boxed{x = 0} \quad 2x - 1 = 0 \quad x + 7 = 0$

$\boxed{x = \frac{1}{2}} \quad \boxed{x = -7}$

c) $f(x) = x^3 - 5x^2 - 4x + 20$

$0 = x^2(x - 5) - 4(x - 5)$

$0 = (x^2 - 4)(x - 5)$

$0 = x^2 - 4 \quad 0 = x - 5$

$x^2 = 4 \quad \boxed{x = 5}$

$\boxed{x = \pm 2}$

d) $f(x) = 4x^3 - 60x^2$

$0 = 4x^2(x - 15)$

$4x^2 = 0 \quad x - 15 = 0$

$\boxed{x = 0} \quad \boxed{x = 15}$

9) Add or subtract the rational functions below:

a) $\frac{x^2+3x-2}{x^2+3x-10} + \frac{4x+12}{x^2+3x-10}$

$\frac{x^2 + 7x + 10}{x^2 + 3x - 10}$

$\frac{(x+2)\cancel{(x+5)}}{(x-2)\cancel{(x+5)}} = \boxed{\frac{x+2}{x-2}}$

b) $\frac{x^2-2x+3}{x^2+7x+12} + \frac{-(x^2+4x+5)}{x^2+7x+12}$

$\frac{2x + 8}{x^2 + 7x + 12}$

$\frac{2\cancel{(x+4)}}{(x+3)\cancel{(x+4)}} = \boxed{\frac{2}{x+3}}$

c) $\frac{3y^2}{3y^2} \cdot \frac{1}{18x^3y} + \frac{5}{27x^2y^3} \cdot \frac{2x}{2x}$

$\boxed{\frac{3y^2 + 10x}{54x^3y^3}}$

d) $\frac{6}{x-2} + \frac{x+3}{2-x} \cdot \frac{(-1)}{(-1)}$

$\frac{6}{x-2} + \frac{-x-3}{x-2}$

$\boxed{\frac{3-x}{x-2}}$

e) $\frac{5}{5} \cdot \frac{5}{x^2-5x} - \frac{x}{5x-25} \cdot \frac{x}{x}$

$5x(x-5) \quad 5(x-5)x$

$\frac{25 - x^2}{5x(x-5)} = \frac{-1(x^2-25)}{5x(x-5)}$

$= \frac{-1(x+5)\cancel{(x-5)}}{5x\cancel{(x-5)}}$

$= \boxed{\frac{-x-5}{5x}}$

f) $\frac{(x+3)}{(x+3)} \cdot \frac{x+2}{x-7} - \frac{x^2+4x+13}{x^2-4x-21}$

$(x+3)(x-7)$

$\frac{x^2 + 5x + 6 + -(x^2 + 4x + 13)}{(x+3)(x-7)}$

$\frac{\cancel{x-7}}{(x+3)\cancel{(x-7)}}$

$= \boxed{\frac{1}{x+3}}$

Find the domain, asymptotes and holes for each rational function if they exist. Then sketch the graph.

10) $f(x)=\dfrac{3x^2+1}{x^2-x+6}=\dfrac{3x^2+1}{(x+2)(x-3)}$

H. Asymp: $\dfrac{3x^2}{x^2} \Rightarrow y=3$

V. Asymp: $x=-2,\ x=3$

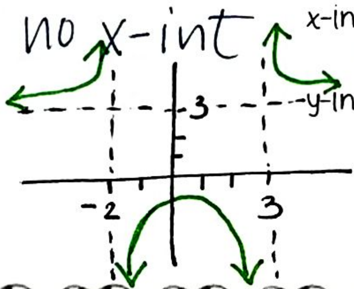
Slant Asymp: none

Hole: none

Domain: $\mathbb{R},\ x\neq -2, 3$

x-intercept: $y=0$ $\quad 3x^2+1=0 \quad \cancel{3x^2=-1}$ no x-int

y-intercept: $x=0$ $\quad \left(0, \frac{1}{6}\right)$



11) $f(x)=\dfrac{x^2+3x}{2x^2-18}=\dfrac{x(x+3)}{2(x^2-9)}=\dfrac{x\cancel{(x+3)}}{2\cancel{(x+3)}(x-3)}$

H. Asymp: $\dfrac{x^2}{2x^2} \Rightarrow y=\frac{1}{2}$

V. Asymp: $x=3$

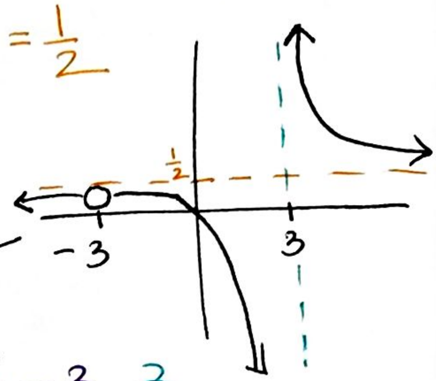
Slant Asymp: none

Hole: $x=-3$

Domain: $\mathbb{R},\ x\neq -3, 3$

x-intercept: $y=0$ $\quad x=0 \quad (0,0)$

y-intercept: $x=0$ $\quad (0,0)$



12) $f(x)=\dfrac{x^2-x}{x+1}$ $\quad =0$

H. Asymp: none

V. Asymp: $x=-1$

Slant Asymp: $y=x-2$

Hole: none

Domain: $\mathbb{R},\ x\neq -1$

x-intercept: $y=0$ $\quad x^2-x=0 \quad x(x-1)=0 \quad x=0 \quad x=1 \quad (0,0)\ (1,0)$

y-intercept: $x=0$ $\quad (0,0)$

$$\begin{array}{r|rrr} -1 & 1 & -1 & 0 \\ & \downarrow & -1 & 2 \\ \hline & 1 & -2 & \boxed{2} \end{array}$$

$y=x-2$

13) $f(x)=\dfrac{\cancel{(x-2)}(2x^2+5x-2)}{x^2-3x+2}$ $\quad \cancel{(x-2)}(x-1)$

$$\begin{array}{r|rrr} 1 & 2 & 5 & -2 \\ & \downarrow & 2 & 7 \\ \hline & 2 & 7 & 5 \end{array}$$

H. Asymp: none

V. Asymp: $x=1$

Slant Asymp: $y=2x+7$

Hole: $x=2$

Domain: $\mathbb{R},\ x\neq 1, 2$

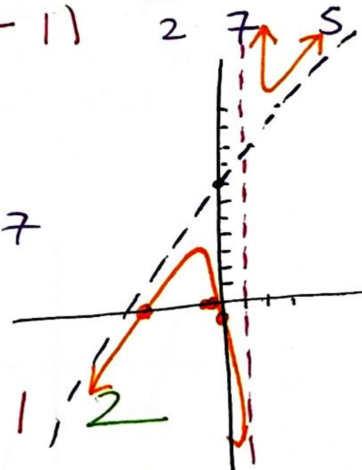
x-intercept: $y=0$ $\quad (.35, 0)\ (-2.85, 0)$ $\quad 2x^2+5x-2=0$

$x=\dfrac{-5\pm\sqrt{5^2-4(2)(-2)}}{2(2)}$

$x=\dfrac{-5\pm\sqrt{41}}{4}$

$x=.35, -2.85$

y-intercept: $x=0$ $\quad -\frac{2}{2}=-1 \quad (0,-1)$



14) Multiply or divide the rational functions below. Leave your answer in simplified form.

a)

$$-(x^2-9x+20)$$

$$\frac{1}{x-4}\cdot\frac{-x^2+9x-20}{x+2}$$

$$\frac{1}{\cancel{x-4}}\cdot\frac{-\cancel{(x-4)}(x-5)}{x+2}$$

$$=\boxed{\frac{-(x-5)}{x+2}}$$

b)

$$\frac{1}{7b+35}\div\frac{3b}{b^2+11b+30}$$

$$7(b+5) \quad (b+5)(b+6)$$

$$\frac{1}{7\cancel{(b+5)}}\cdot\frac{\cancel{(b+5)}(b+6)}{3b}$$

$$=\boxed{\frac{b+6}{21b}}$$

c)

$$\frac{1}{8-5x}\div\frac{5x^2}{15x^2-24x}$$

$$-1(5x-8) \quad 3x(5x-8)$$

$$\frac{1}{-\cancel{(5x-8)}}\cdot\frac{3\cancel{x}\cancel{(5x-8)}}{5x^{\cancel{2}}}=\frac{3}{-5x}$$

$$=\boxed{-\frac{3}{5x}}$$

d)

$$7(7x+5) \qquad (5x-8)(7x+5)$$

$$(49x+35)\div\frac{35x^2-31x-40}{5x-8}$$

$$\frac{7\cancel{(7x+5)}}{1}\cdot\frac{\cancel{5x-8}}{\cancel{(5x-8)}\cancel{(7x+5)}}$$

$$=\boxed{7}$$

15) Match each graph below with its equation. Remember what repeating zeros look like, characteristics of leading coefficients, and degree of functions to help you. (Try to do this without your calculator!)

21. $f(x)=x^2(x-2)(x-4)$

22. $f(x)=x^2(x-3)^3$

23. $f(x)=-3(x-1)(x-2)^2(x-3)$

24. $f(x)=4x^2-9$

26. $f(x)=-2x^3+8x$

27. $f(x)=(x-1)(x-3)(x-5)$

28. $f(x)=x^4-3x^3$

29. $f(x)=-(x-4)(x-3)(x-1)^2$

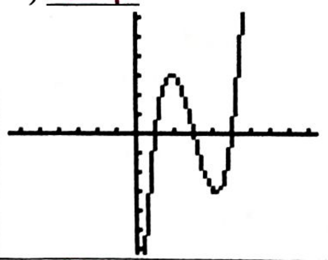
31. $f(x)=-2(x+3)^2(x+1)^2$

32. $f(x)=-x^3+9x \quad -x(x^2-9)$

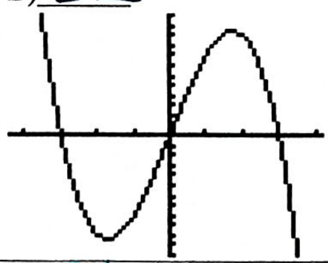
33. $f(x)=3x^4-3x^3-3x^2+3x$

34. $f(x)=x^3-3x^2 \quad x^2(x-3)$

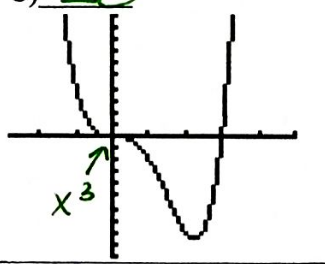
A) 27



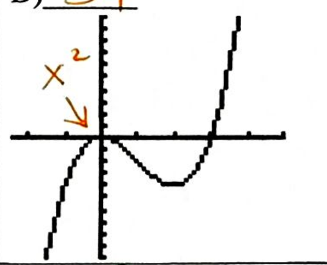
B) 32



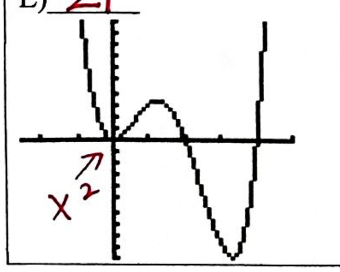
C) 28



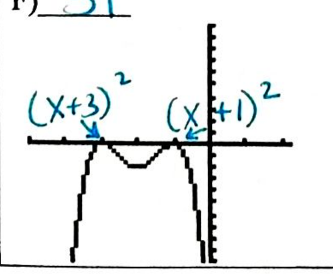
D) 34



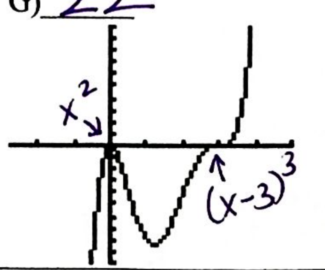
E) 21



F) 31



G) 22



H) 23