Trigonometry/Precalculus
Chapter 3 Review
Day______ CW/HW

Name
Date
Block

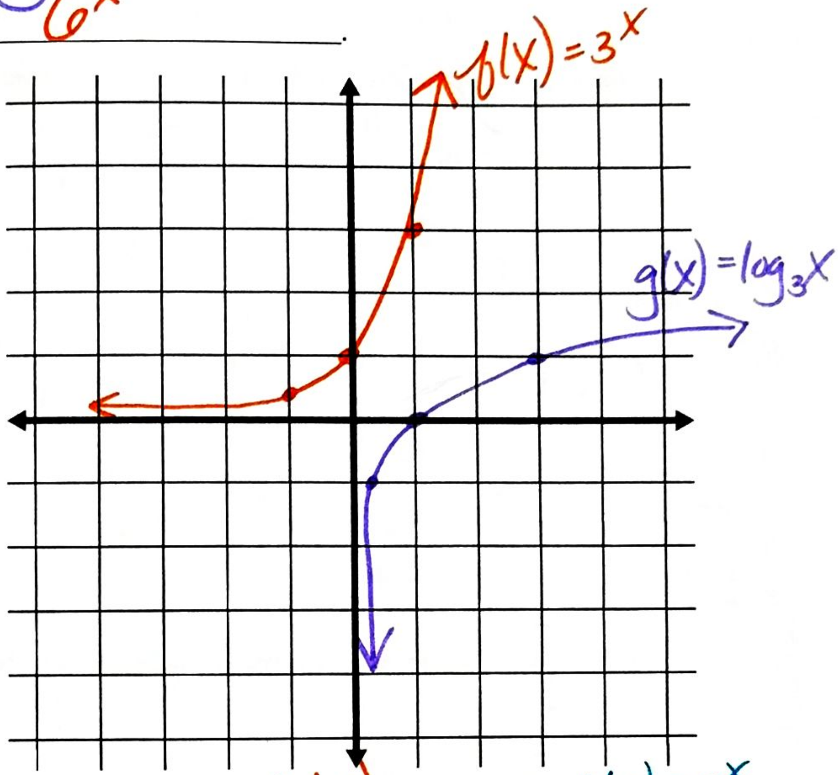
Fill in the following statements.

1) The inverse of an exponential function is a logrithmic function.

2) The inverse of $f(x) = 7^x$ is $g(x) =$ $\log_7 x$.

3) The inverse of $f(x) = \log_6 x$ is $g(x) =$ 6^x.

4) Use the idea of inverses to draw an accurate sketch of both $f(x) = 3^x$ and $g(x) = \log_3 x$. Then state the domain, range and asymptote of each.

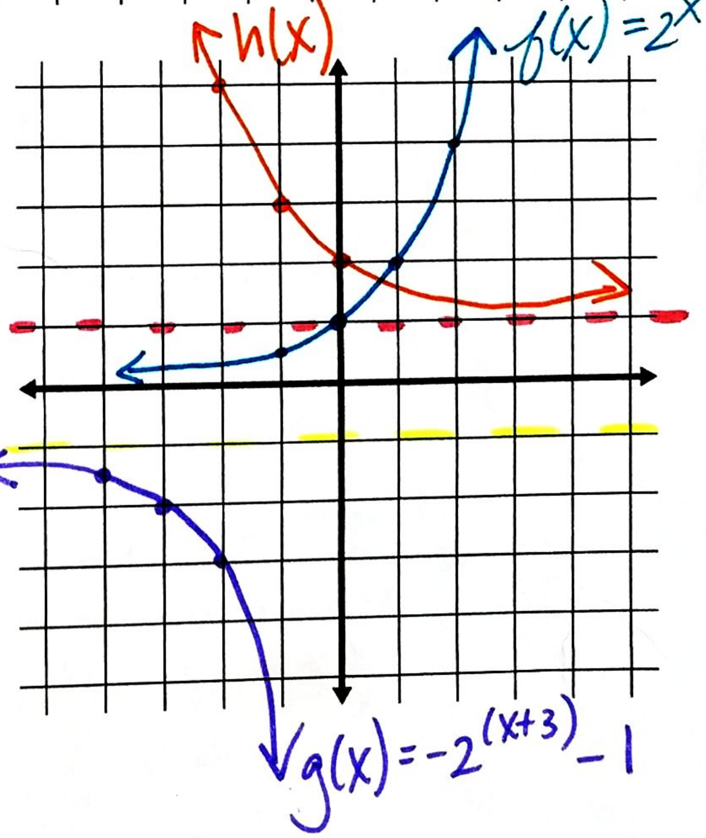


$f(x)$	$g(x)$
D: $\mathbb{R}$	D: $(0,\infty)$
R: $(0,\infty)$	R: $\mathbb{R}$
Asymptote: $y=0$	Asymptote: $x=0$

5 a) Draw and accurate sketch of $f(x) = 2^x$.

b) Describe how $g(x) = -2^{(x+3)} - 1$ compares to $f(x)$.

- flipped over x-axis
- shifted 3 left, down 1



c) Use part b to draw $g(x)$.

d) Sketch and label $h(x) = 2^{-x} + 1$

6*) Arrange the following in order from smallest to largest. (try without a calculator)

$\log_2 5 \approx 2.2$ $\quad \log_2 1 = 0$ $\quad \log_2 3 \approx 1.5$

$\log_3 9 = 2$ $\quad \log_3 8 \approx 1.9$ $\quad \log_2 \frac{1}{2} = -1$

$\log_3 \sqrt[3]{3} = \frac{1}{3}$ $\quad \log_2 8 = 3$

a) $\log_2 \frac{1}{2}$ e) $\log_3 8$

b) $\log_2 1$ f) $\log_3 9$

c) $\log_3 \sqrt[3]{3}$ g) $\log_2 5$

d) $\log_2 3$ h) $\log_2 8$

7*) Find the exact value of each logarithmic expression without using a calculator.

a) $\log_4 64 = X$

$4^X = 64$

$4^X = 4^3$

$\boxed{X = 3}$

b) $\log_2 \sqrt[4]{8} = X$

$2^X = \sqrt[4]{8}$

$2^X = \sqrt[4]{2^3}$

$2^X = 2^{3/4}$

$X = \boxed{3/4}$

c) $\log_4 16^{1.2}$

$\log_4 (4^2)^{1.2}$

$\cancel{\log_4} 4^{2.4}$

$\boxed{2.4}$

d) $\ln \frac{1}{\sqrt{e}}$

$\ln \frac{1}{e^{1/2}}$

$\cancel{\ln} e^{-\frac{1}{2}} = \boxed{-\frac{1}{2}}$

e) $\log_9 \frac{1}{3} = X$

$9^X = \frac{1}{3}$

$(3^2)^X = 3^{-1}$

$2X = -1$ $\quad X = \boxed{-\frac{1}{2}}$

f) $\log \frac{1}{1000}$

$\log \frac{1}{10^3}$

$\cancel{\log 10}^{-3}$

$\boxed{-3}$

g) $\log_8 1 = X$

$8^X = 1$

$X = \boxed{0}$

h) $\ln e^3 - \ln e^7$

$\ln \frac{e^3}{e^7}$

$\cancel{\ln} e^{-4} = \boxed{-4}$

8) Write an equation in the form $y = ab^x$ that passes through the following points (1, 12) and (-1, 3).

↑ ↑
x y

$12 = a \cdot b^1$ $\quad 3 = a \cdot b^{-1}$

$$\frac{12 = a \cdot b^1}{3 = a \cdot b^{-1}}$$

$4 = b^{1-(-1)}$

$4 = b^2$ $\quad b = 2$

$$\frac{12}{2} = \frac{a \cdot 2^1}{2}$$

$a = 6$

$\boxed{y = 6(2)^x}$

9) Use the properties of logs to expand the following expressions:

a) $\log xyz^2$

$\log x + \log y + \log z^2$

$\boxed{\log x + \log y + 2\log z}$

b) $\log \frac{6}{\sqrt{x+1}} = \log 6 - \log \sqrt{x+1}$

$\log 6 - \log (x+1)^{\frac{1}{2}}$

$\boxed{\log 6 - \frac{1}{2}\log(x+1)}$

10) If $\log_b M = 2.1$ and $\log_b N = -3$, use properties of logs to find $\log_b \frac{\sqrt{M}}{N^3}$.

$\log_b \sqrt{M} - \log_b N^3$

$\log_b M^{\frac{1}{2}} - \log_b N^{3}$

$\frac{1}{2}\log_b M - 3\log_b N$

$\frac{1}{2}(2.1) - 3(-3)$

$1.05 + 9$

$\boxed{10.05}$

11) Solve the following. Find exact values when possible.

a) $\log_5 x = 3$

$X = 5^3$

$\boxed{X = 125}$

b*) $9^{x+10} = 27^{2x+3}$

$(3^2)^{X+10} = (3^3)^{2X+3}$

$2(X+10) = 3(2X+3)$

$2X + 20 = 6X + 9$

$11 = 4X$

$\boxed{X = 11/4}$

c) $7 + \log(x-4) = 9$

$-7 \qquad -7$

$\log_{10}(X-4) = 2$

$10^2 = X - 4$

$100 = X - 4$

$\boxed{X = 104}$

d) $\frac{243}{3} = \frac{3(10^x)}{3}$

$\log 81 = \log 10^x$

$X = \log 81$

$\boxed{X = 1.91}$

e) $e^{x-3} + 8 = 10$

$-8 \quad -8$

$\ln e^{X-3} = \ln 2$

$X - 3 = \ln 2$

$+3 \qquad +3$

$X = \ln 2 + 3$

$\boxed{X = 3.69}$

f) $\log_5 x - \log_5 7 = \log_5 12$

$\log_5 \frac{X}{7} = \log_5 12$

$\frac{X}{7} = \frac{12}{1}$

$X = 12(7)$

$\boxed{X = 84}$

g) ½ log x + log 5 = log 30

$\log X^{\frac{1}{2}} + \log 5 = \log 30$

$\log 5X^{\frac{1}{2}} = \log 30$

$5X^{\frac{1}{2}} = 30$

$(X^{\frac{1}{2}})^2 = (6)^2$

$\boxed{X = 36}$

h) $\log_2 x - \log_2 3 = 4$

$\log_2 \frac{X}{3} = 4$

$2^4 = \frac{X}{3}$

$\frac{16}{1} = \frac{X}{3}$

$\boxed{X = 48}$

i) $\ln \frac{2}{x} = 1$

$e^1 = \frac{2}{X}$

$\frac{eX}{e} = \frac{2}{e}$

$\boxed{X = \frac{2}{e}}$

j) $\log_3 (5x^2) = \log_3 (3x+2)$

$5X^2 = 3X + 2$

$-3X - 2 \qquad -3X - 2$

$5X^2 - 3X - 2 = 0$

$(5X+2)(X-1) = 0$

$5X + 2 = 0$

$5X = -2$

$\boxed{X = -\frac{2}{5}}$

$X - 1 = 0$

$\boxed{X = 1}$

k) $\log_x 2{,}401 = 4$

$\sqrt[4]{X^4} = \sqrt[4]{2401}$

$\boxed{X = 7}$

l*) $\left(\frac{1}{32}\right)^x = 16^{(x+1)}$

$\left(\frac{1}{2^5}\right)^X = 2^{4(X+1)}$

$(2^{-5})^X = 2^{4(X+1)}$

$-5X = 4(X+1)$

$-5X = 4X + 4$

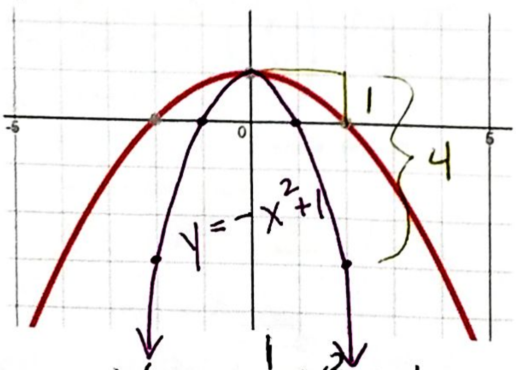
$-4X \qquad -4X$

$\frac{-9X}{-9} = \frac{4}{-9}$

$\boxed{X = -\frac{4}{9}}$

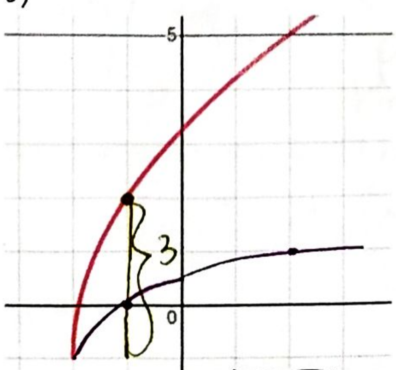
12*) Write the equation of each transformed parent function.

a)



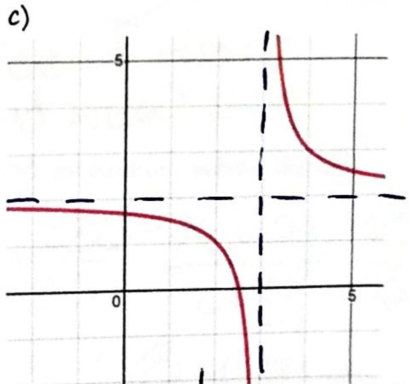
$Y = -\frac{1}{4}X^2 + 1$

b)



$Y = 3\sqrt{X+2} - 1$

c)



$Y = \frac{1}{X-3} + 2$

13) **In a research experiment, a population of fruit flies is increasing according to the law of exponential growth. After 2 days there are 100 flies, and after 4 days there are 300 flies.**

$(2, 100)$
$(4, 300)$

a) Using the equation $A = Pe^{kt}$ where t is time in days, find the growth constant k for the fruit fly population.	b) Write an equation to represent the fruit fly population at any time t after the experiment has started.
$\frac{300=Pe^{4K}}{100=Pe^{2K}}$ $\boxed{K=.549}$ $\ln 3 = \ln e^{2K}$ $\ln 3 = 2K$	$\frac{100=Pe^{2(.549)}}{e^{2(.549)}\quad e^{2(.549)}}$ $P=33.4$ $\boxed{A=33.4e^{.549t}}$
c) Find the population after 5 days.	d) After how many days will the fly population reach 1000?
$A=33.4e^{.549(5)}$ $A=519.9$ $\approx \boxed{520 \text{ fruit flies}}$	$\frac{1000}{33.4}=\frac{33.4e^{.549t}}{33.4}$ $\boxed{t=6.2 \text{ days}}$ $\ln 29.94 = \ln e^{.549t}$ $\frac{\ln 29.94}{.549}=\frac{.549t}{.549}$

14) Chromium-48 has a short half-life of 21.6 hours. How long will it take 360 g of chromium-48 to decay to 11.25 g?

$A=P\left(\frac{1}{2}\right)^{t/h} \rightarrow \frac{11.25}{360}=\frac{360\left(\frac{1}{2}\right)^{t/21.6}}{360}$

$\log .03125 = \log \frac{1}{2}^{t/21.6}$

$\frac{\log .03125}{\log \frac{1}{2}} = \frac{\frac{t}{21.6}\log\frac{1}{2}}{\log\frac{1}{2}}$

$5=\frac{t}{21.6}$

$\boxed{t=108\text{hrs}}$

15) A 208 g sample of sodium-24 decays to 13.0 g within 60.0 hours. What is the half-life of this radioactive isotope?

$\frac{13}{208}=\frac{208\left(\frac{1}{2}\right)^{60/h}}{208}$

$\log .0625 = \log \frac{1}{2}^{60/h}$

$\frac{\log .0625}{\log .5} = \frac{\frac{60}{h}(\log .5)}{\log .5}$

$4=\frac{60}{h}$

$4h=60$

$\boxed{h=15\text{hours}}$

16) **A year after the purchase of a new car, its value is appraised at \$18,000. Four years after its purchase, the car's value is \$11, 054.**

$(1, 18{,}000)$
$(4, 11{,}054)$

a) Write an equation in the form $y = ab^x$ that represents the value of the car after its purchase.

$\frac{11{,}054=a\cdot b^4}{18{,}000=a\cdot b^1}$

$(.61411)^{\frac{1}{3}}=(b^3)^{\frac{1}{3}}$

$b=.85$

$18{,}000=a(.85)$

$a=21{,}176.47$

b) What does the value of "a" in your equation represent?

$\boxed{y=21{,}176.47(.85)^x}$

price of the brand new car

c) If the car's owner wants to sell the car when its value is still \$5,000 or higher, when should he sell the car?

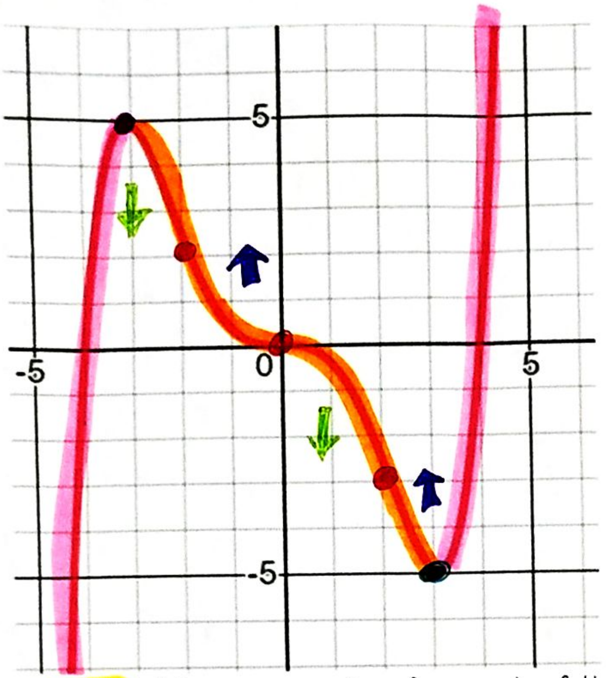
$\frac{5000}{21{,}176.47}=\frac{21{,}176.47(.85)^x}{21{,}176.47}$

$\log .236 = \log .85^x$

$\frac{\log .236}{\log .85}=\frac{x\log .85}{\log .85}$

$x=8.88$ years

$\boxed{\text{before 8.9 years of ownership}}$



17) Determine each of the following for the given polynomial function given at points of inflection at (-2, 2), (0, 0) and (2, -3).

Relative Minimum(s): $(3, -5)$

Relative Maximum(s): $(-3, 5)$

Intervals of decrease: $(-3, 3)$

Interval(s) of increase: $(-\infty, -3) \cup (3, \infty)$

Concave Up Interval(s): $(-3, 0) \cup (2, \infty)$

Concave Down Interval(s): $(-\infty, -2) \cup (0, 2)$

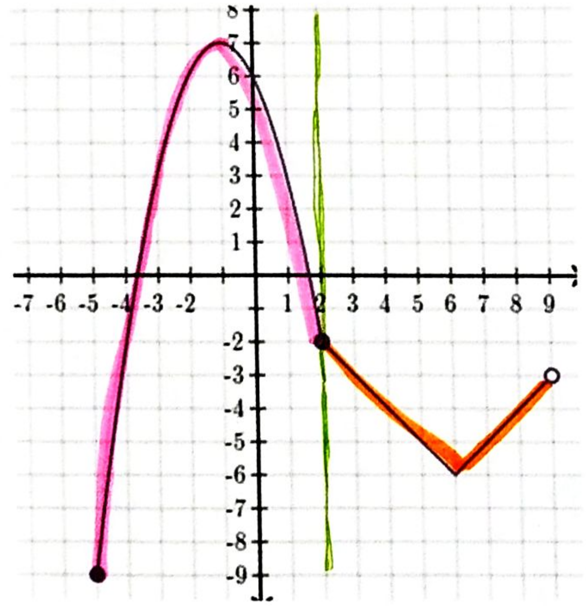
18*) Write an equation for each of the following piecewise functions.

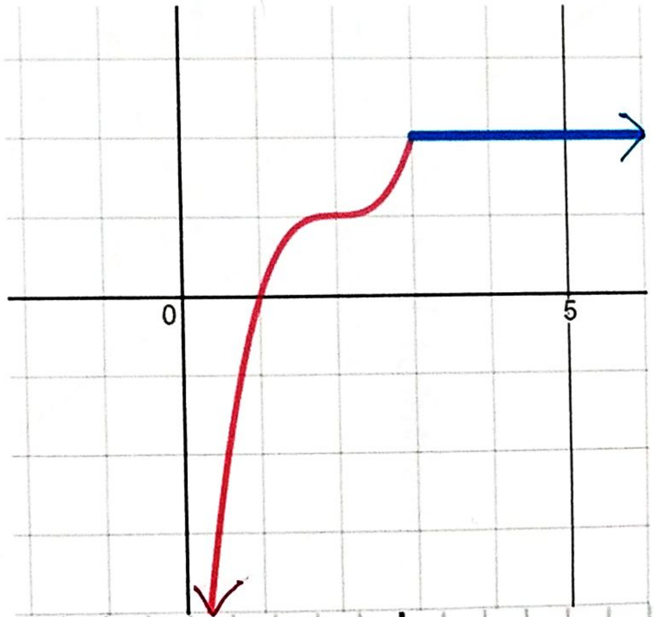
a)

$$f(x) = \begin{cases} -(x+1)^2 + 7 & -5 \le x \le 2 \\ |x-6| - 6 & 2 < x < 9 \end{cases}$$

b)

$$f(x) = \begin{cases} (x-2)^3 + 1 & x \le 3 \\ 2 & x > 3 \end{cases}$$





19*) Make a careful sketch of the piecewise function below:

$$f(x) = \begin{cases} -\frac{1}{2}x + 3 & x \le -2 \\ -(x+1)^3 + 3 & -2 < x < 0 \\ |x-2| & x \ge 0 \end{cases}$$