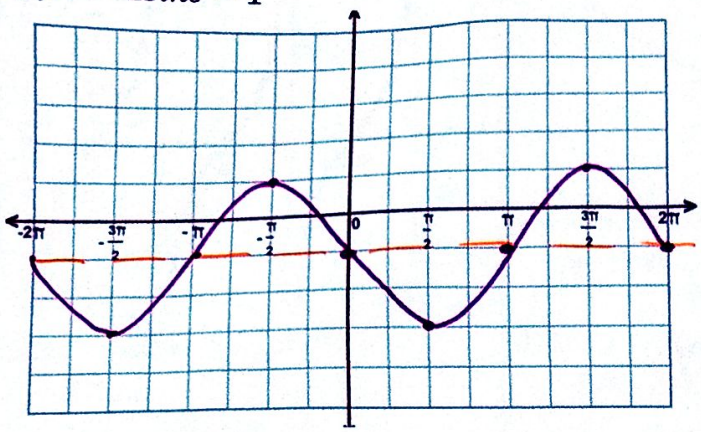
Graph 2 cycles (if possible) of each of the following functions:

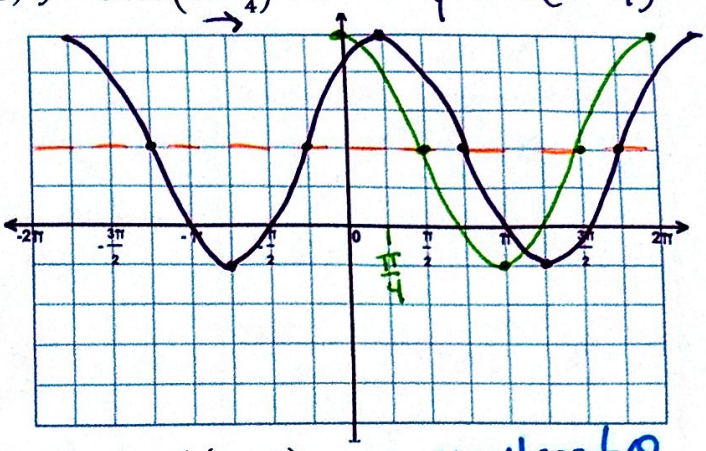
24) $y = -2\sin\theta - 1$



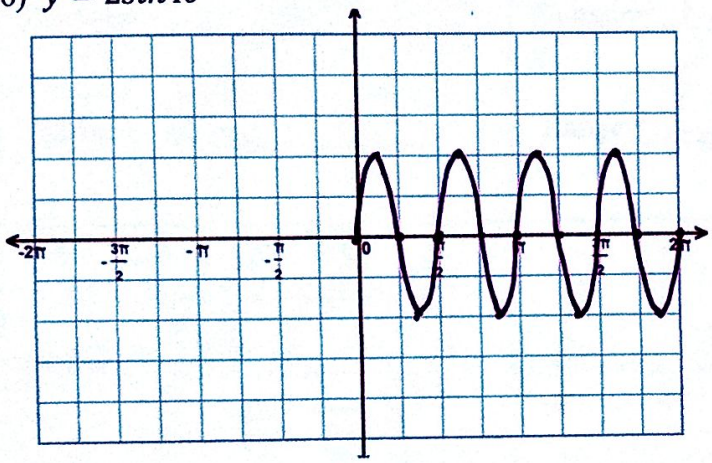
25) $y = 3\cos\left(\theta - \frac{\pi}{4}\right) + 2$

$y = 3\cos\theta + 2$

$y = 3\cos\left(\theta - \frac{\pi}{4}\right) + 2$



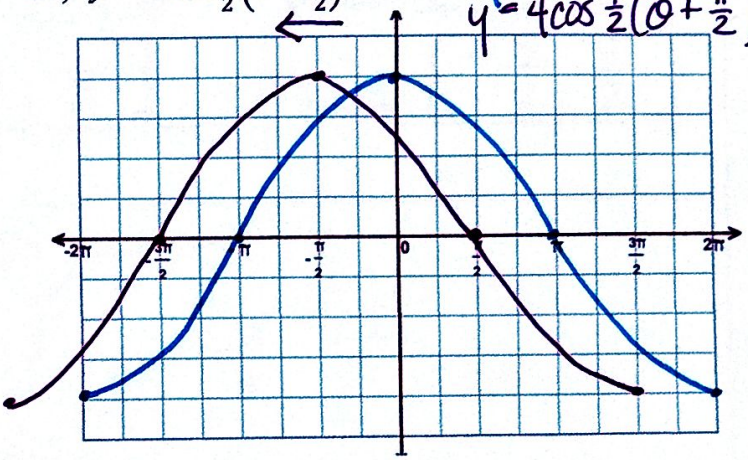

26) $y = 2\sin 4\theta$



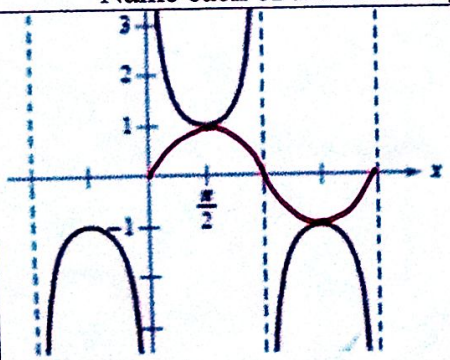
27) $y = 4\cos\frac{1}{2}\left(\theta + \frac{\pi}{2}\right)$

$y = 4\cos\frac{1}{2}\theta$

$y = 4\cos\frac{1}{2}\left(\theta + \frac{\pi}{2}\right)$



Name each of the following functions. Then state the domain and range for each.

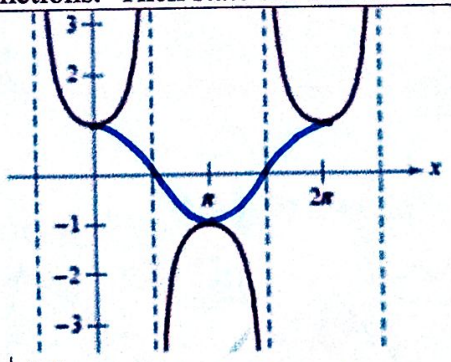


28) y = csc x

Domain= $\mathbb{R}, x \neq \pi n$

Range= $(-\infty, -1]$ and $[1, \infty)$

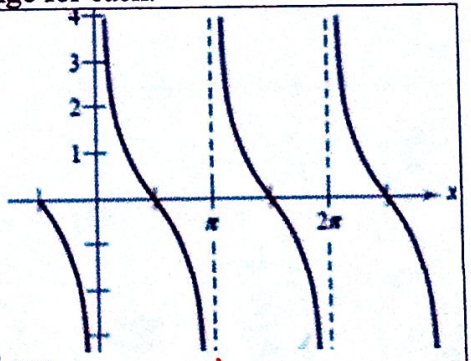
$(-\infty, -1] \cup [1, \infty)$



29) y = sec x

Domain= $\mathbb{R}, x \neq \frac{\pi}{2} + \pi n$

Range= $(-\infty, -1]$ and $[1, \infty)$



30) y = cot x

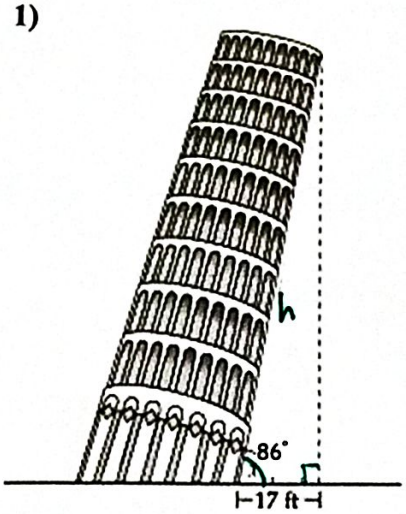
Domain= $\mathbb{R}, x \neq \pi n$

Range= $\mathbb{R}$

Trigonometry/Precalculus
Chapter 4 Test Review
Day ____

Name
Date
Block

1)




The Leaning Tower of Pisa now leans at an 86° degree angle with the horizontal. How tall would the Tower be if it did not lean?

$\cos 86 = \frac{17}{h}$

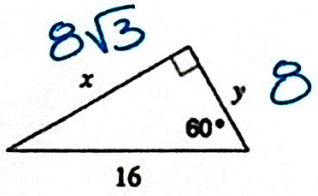
$.0698 = \frac{17}{h}$

$.0698h = 17$

$h = 243.7 ft$

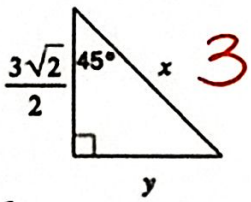
2) Find the missing side of each triangle. Leave each value in simplified radical form when applicable.

a)



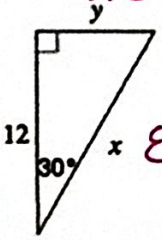

$8\sqrt{3}$ $8$

b)




$3$ $\left(\frac{3\sqrt{2}}{2}\right)\sqrt{2}$ $\frac{3\sqrt{2}}{2}$

c)




$4\sqrt{3}$ $8\sqrt{3}$ $\frac{12}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{12\sqrt{3}}{3} = 4\sqrt{3}$

3) Rationalize the denominator for each expression. Simplify completely.

a) $\frac{6}{5\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}$

$\frac{6\sqrt{2}}{5\sqrt{4}} = \frac{6\sqrt{2}}{5\cdot 2} =$

$\frac{6\sqrt{2}}{10} = \frac{3\sqrt{2}}{5}$

b) $\frac{4}{2\sqrt{3}-8}\frac{(2\sqrt{3}+8)}{(2\sqrt{3}+8)}$

$\frac{8\sqrt{3}+32}{12-64}$

$\frac{8\sqrt{3}+32}{-52} = \boxed{\frac{2\sqrt{3}+8}{-13}}$

| $2\sqrt{3}$ | $-8$ |
|---|---|
| $2\sqrt{3}$ | $4\cdot 3$ / $12$ |
| $8$ | $-64$ |

c) $\frac{10-\sqrt{5}}{2+\sqrt{5}}\frac{(2-\sqrt{5})}{(2-\sqrt{5})}$

| $10$ | $-\sqrt{5}$ |
|---|---|
| $2$ | $20$ / $-2\sqrt{5}$ |
| $-\sqrt{5}$ | $-10\sqrt{5}$ / $5$ |

$\frac{25-12\sqrt{5}}{-1}$

| $2$ | $\sqrt{5}$ |
|---|---|
| $4$ | $2\sqrt{5}$ |
| $-\sqrt{5}$ | $-2\sqrt{5}$ / $-5$ |

$= \boxed{-25+12\sqrt{5}}$

Convert the following from degrees to radians.

4) $368 \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{5}}$

5) $-320^\circ$

$-16 \div 20$

$-320 \cdot \frac{\pi}{180} \div 20 = \boxed{\frac{-16\pi}{9}}$

Convert the following from radians to degrees.

6) $\frac{\pi}{15} \cdot \frac{180}{\pi} = \boxed{12^\circ}$

7) $-\frac{17\pi}{20} \cdot \frac{180}{\pi} = \boxed{153^\circ}$

Use a calculator to find the value of secant, cosecant and cotangent of 115°. Write the ratio you used and round to 3 decimal places.

**8)** sec 115°

$\frac{1}{\cos 115} = -2.366$

**9)** csc 115°

$\frac{1}{\sin 115^\circ} = 1.103$

**10)** cot 115°

$\frac{1}{\tan 115^\circ} = -.466$

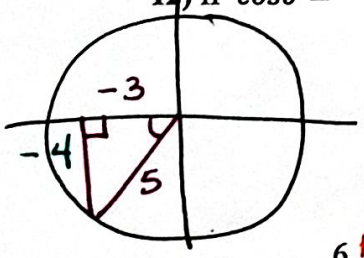
11) Find the **exact** value of the six trigonometric functions for each of the given values. Remember to simplify each value completely.

coterm.

| | a) $\frac{5\pi}{6}$ | b) $-\frac{5\pi}{3}$ $\frac{\pi}{3}$ | c) $\frac{5\pi}{4}$ |
|---|---|---|---|
| sin | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{2}}{2}$ |
| cos | $-\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ |
| tan | $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | $\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$ |
| csc | $-2$ | $\frac{2\sqrt{3}}{3}$ | $-\frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{-2\sqrt{2}}{\sqrt{4}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$ |
| sec | $-\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{-2\sqrt{3}}{3}$ | $2$ | $-\sqrt{2}$ |
| cot | $-\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ | $1$ |

cos −  sin −

12) If $\cos\theta = -\frac{3}{5}$ and $\sin\theta < 0$, find the exact values of the other five trig functions. Show your work.




$\sin\theta = -\frac{4}{5}$ $\quad \csc\theta = -\frac{5}{4}$

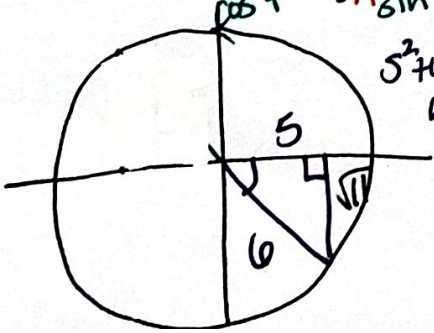
$\cos\theta = -\frac{3}{5}$ $\quad \sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

13) If $\sec\theta = \frac{6}{5}$, $\tan\theta < 0$, find the exact values of the other five trig functions. Show your work.

H

cos +  A  sin −




$5^2 + y^2 = 6^2$

$y^2 = 11$

$y = \sqrt{11}$

$\sin\theta = \frac{\sqrt{11}}{6}$ $\quad \csc\theta = \frac{6}{\sqrt{11}}\left(\frac{\sqrt{11}}{\sqrt{11}}\right) = \frac{6\sqrt{11}}{11}$

$\cos\theta = \frac{5}{6}$ $\quad \sec\theta = \frac{6}{5}$

$\tan\theta = \frac{\sqrt{11}}{5}$ $\quad \cot\theta = \frac{5}{\sqrt{11}}\left(\frac{\sqrt{11}}{\sqrt{11}}\right) = \frac{5\sqrt{11}}{11}$

cot −
sin +
sin +
cot −

**14)** a) State the quadrant in which the terminal side of θ lies if $\cot\theta < 0$ and $\sin\theta > 0$.

Quadrant II

**15)b)** State the quadrant in which the terminal side of θ lies if $\csc\theta < 0$ and $\tan\theta > 0$.

Quadrant III

tan +
sin −
sin −
tan +

**16)** Are $\frac{29\pi}{4}$ and $\frac{5\pi}{4}$ coterminal angles? Explain why or why not.

$2\pi = \frac{8\pi}{4}$

$\frac{29\pi}{4} - \frac{8\pi}{4} - \frac{8\pi}{4} - \frac{8\pi}{4} = \boxed{\frac{5\pi}{4}}$ → Yes, because when you subtract $2\pi$ three times you get $\frac{5\pi}{4}$.

**17)** Find the sine of each angle below. (Use coterminal angles to help you.)

$2\pi = \frac{6\pi}{3}$

**a)** $\sin\frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$

$\sin\frac{5\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$

**b)** $\sin\frac{21\pi}{6} - \frac{12\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$

$2\pi = \frac{12\pi}{6}$

$\sin\frac{3\pi}{2} = \boxed{-1}$

**18)** Write an equation for a cosine function shifted up 4 and right $\frac{\pi}{3}$, with a period of $\frac{3\pi}{4}$ and amplitude 7.

$k = 4$, $h = \frac{\pi}{3}$, $p = \frac{3\pi}{4}$, $a = 7$

$pb = 2\pi$

$\left(\frac{3\pi}{4}\right)b = (2\pi)\frac{4}{3\pi}$

$\frac{4}{3\pi}$ $b = \frac{8}{3}$

$y = a\cos b(x-h)+k$

$\boxed{y = 7\cos\frac{8}{3}\left(x-\frac{\pi}{3}\right)+4}$

**19)** Write an equation for a sine function with amplitude 4, shifted left $\frac{\pi}{6}$ with period of $\frac{2\pi}{3}$.

$a = 4$, $h = -\frac{\pi}{6}$, $p = \frac{2\pi}{3}$

$pb = 2\pi$

$\frac{3}{2\pi}\left(\frac{2\pi}{3}b\right) = (2\pi)\frac{3}{2\pi}$

$b = 3$

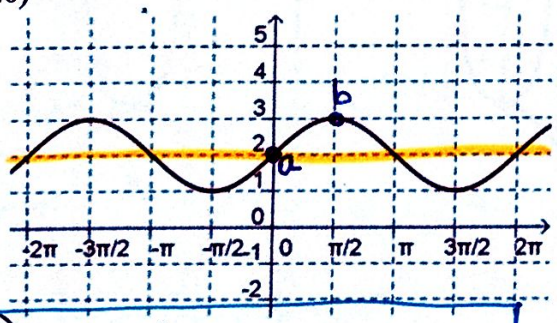
$y = a\sin b(x-h)+k$

$y = 4\sin 3\left(x-\frac{-\pi}{6}\right)$

$\boxed{y = 4\sin 3\left(x+\frac{\pi}{6}\right)}$

Write an equation for each sinusoidal curve in terms of sine and cosine using the indicated points.

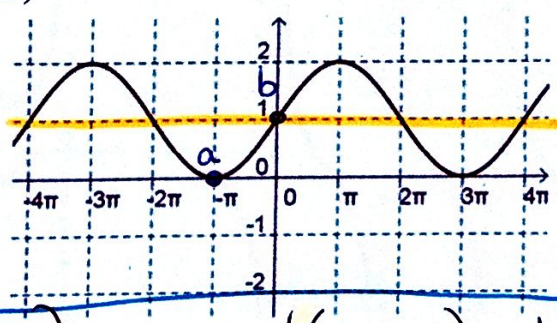
**20)**




a) $y = \sin x + 2$

b) $y = \cos\left(x-\frac{\pi}{2}\right)+2$

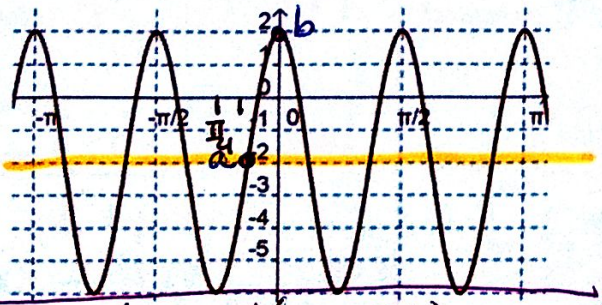
**21)**




a) $y = -\cos\frac{1}{2}(x+\pi)+1$

b) $y = \sin\frac{1}{2}x+1$

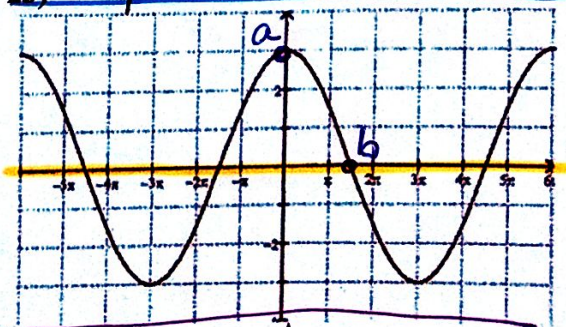
**22)**




a) $y = 4\sin 4\left(x+\frac{\pi}{8}\right)-2$

b) $y = 4\cos 4x - 2$

**23)**




$p = 6\pi$

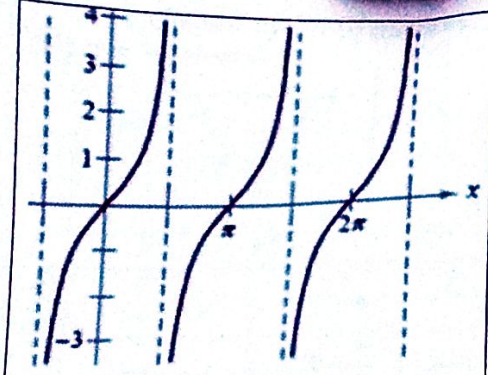
$pb = 2\pi$

$6\pi b = 2\pi$

$6\pi$ $b = \frac{1}{3}$

a) $y = 3\cos\frac{1}{3}x$

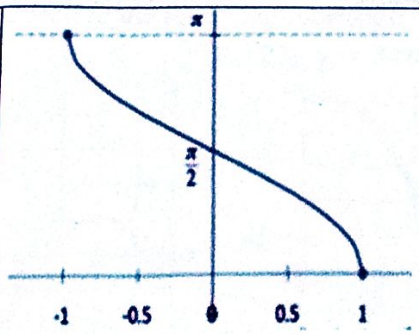
b) $y = -3\sin\frac{1}{3}\left(x-\frac{3\pi}{2}\right)$




31) y = $\tan x$

Domain= $\mathbb{R}, x \neq \frac{\pi}{2} + \pi n$

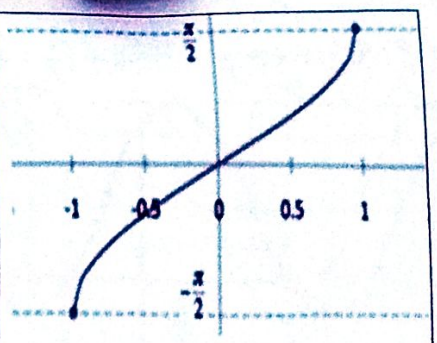
Range= $\mathbb{R}$




32) y = $\cos^{-1} x$

Domain= $[-1, 1]$

Range= $[0, \pi]$




33) y = $\sin^{-1} x$

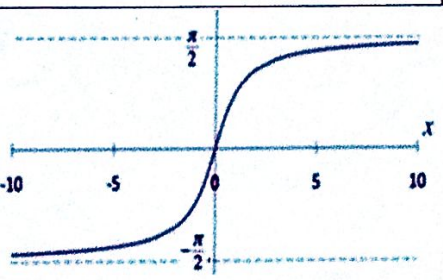
Domain= $[-1, 1]$

Range= $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

34) y = $\tan^{-1} x$

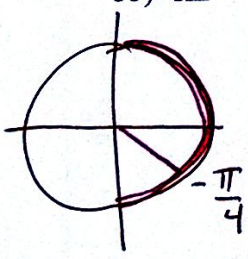
Domain= $\mathbb{R}$

Range= $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

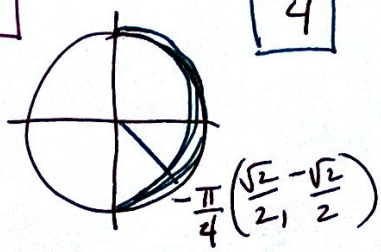



Evaluate each expression:

35) $\sin^{-1} -\frac{\sqrt{2}}{2} = \boxed{-\frac{\pi}{4}}$




36) $\arctan -1 = \boxed{-\frac{\pi}{4}}$




37) $\cos\left(\sin^{-1} -\frac{1}{2}\right)$

$\cos\left(-\frac{\pi}{6}\right)$

$= \boxed{\frac{\sqrt{3}}{2}}$

38) $\csc\left(\cos^{-1} \frac{1}{2}\right)$

$\csc\left(\frac{\pi}{3}\right)$

$= \boxed{\frac{2\sqrt{3}}{3}}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\csc\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

$\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{3}$

Solve each of the following trigonometric equations.

39) $\tan^2 x - 1 = 0$

$+1 \quad +1$

$\sqrt{\tan^2 x} = \sqrt{1}$

$\tan x = \pm 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

40) $(\sec x - 2)(2\cos x - 1) = 0$

$\sec x - 2 = 0$

$\sec x = 2$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$