6.4 Dot Product & Angles Between Vectors

Finding a Dot Product of 2 Vectors

⇒ 3rd vector operation

⇒ helps determine angles between vectors (forces)

$\vec{v} = \langle x_1, y_1 \rangle \quad \vec{w} = \langle x_2, y_2 \rangle$

$$\boxed{\vec{v} \cdot \vec{w} = (x_1)(x_2) + (y_1)(y_2)}$$

outcome of a dot product is a number (not a vector)

* If $\vec{v} \cdot \vec{w} = 0$, then the vectors are orthogonal. (perpendicular)

ex: Determine whether $\vec{v} = \langle 3, -5 \rangle$ and $\vec{w} = \langle 4, 2 \rangle$ are orthogonal or not.

$\vec{v} \cdot \vec{w} = 3(4) + (-5)(2) = 2$ ← since $\vec{v} \cdot \vec{w} \neq 0$, vectors are not orthogonal.

Angles Between Vectors

$$\boxed{\cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}}$$

← $\|\vec{v}\| = \sqrt{x^2 + y^2}$

ex: Determine the angle between $\vec{v} = \langle 3, -5 \rangle$ and $\vec{w} = \langle 4, 2 \rangle$.

$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \leftarrow 2$$

$$\|\vec{v}\| = \sqrt{3^2 + (-5)^2} = \sqrt{34}$$

$$\|\vec{w}\| = \sqrt{4^2 + 2^2} = \sqrt{20}$$

calculator

$$\cos\theta = \frac{2}{\left((\sqrt{34})(\sqrt{20})\right)}$$

$\cos^{-1}$ $\cos\theta = \cos^{-1} .077$

$$\boxed{\theta = 85.6^\circ}$$

* Vectors are parallel if:
- their components are multiples of each other
- their slopes $\left(\frac{y}{x}\right)$ are equal

$$\vec{a} = \langle -3, -2 \rangle$$

$x(-2) \downarrow \quad \downarrow x(-2)$

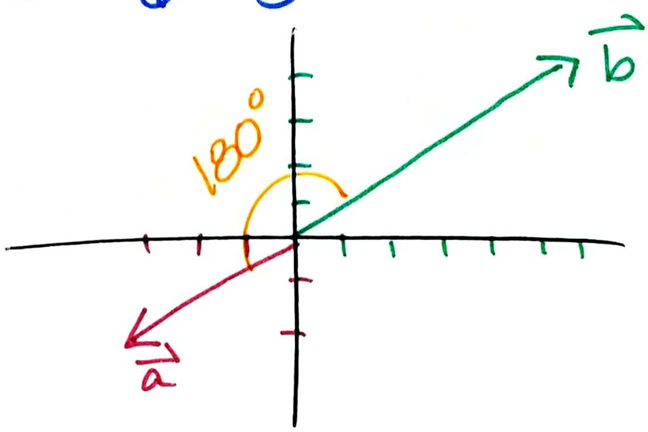
$$\vec{b} = \langle 6, 4 \rangle$$

$$m_{\vec{a}} = \frac{-2}{-3} = \frac{2}{3}$$

$$m_{\vec{b}} = \frac{4}{6} = \frac{2}{3}$$

=

if $\cos\theta = -1$ using formula above, $\theta = 180^\circ$ and vectors are parallel



ex: Determine whether $\vec{u} \parallel \vec{v}$ if

$\vec{u} = \langle \frac{5}{3}, -\frac{1}{6} \rangle$ $\quad \vec{v} = \langle 60, -6 \rangle$

$$m_{\vec{u}} = \frac{-\frac{1}{6}}{\frac{5}{3}}$$

$$m_{\vec{v}} = \frac{-6}{60} = -\frac{1}{10}$$

$$-\frac{1}{2\cancel{6}} \cdot \frac{\cancel{3}}{5}$$

$$= -\frac{1}{10}$$

since the slopes are $=$, $\vec{u} \parallel \vec{v}$

ex: Find y so that $\vec{a}$ and $\vec{b}$ are orthogonal given $\vec{a} = \langle 4, 2 \rangle$ and $\vec{b} = \langle -7, y \rangle$

$$4(-7) + 2y = 0$$

$$-28 + 2y = 0$$

$$2y = 28$$

$$\boxed{y = 14}$$