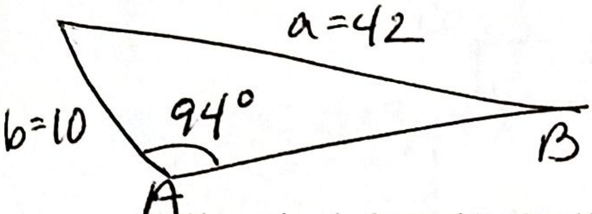
Trigonometry/Precalculus
Fall Semester Final Review
Day ______

Name
Date
Block

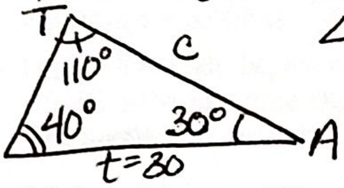
Chapter 6

1) Given triangle ABC with $m\angle A = 94°$, $b = 10$, $a = 42$, solve for <B.



$$\frac{\sin 94}{42} = \frac{\sin B}{10}$$
$$10\sin 94 = 42\sin B$$
$$\sin B = \frac{10\sin 94}{42}$$
$$\sin^{-1} \sin B = .2375 \sin^{-1}$$
$$\boxed{B = 13.7°}$$

2) Given triangle CAT with <C = 40°, <A = 30°, and side t = 30ft, use Law of Sines to solve for side c.

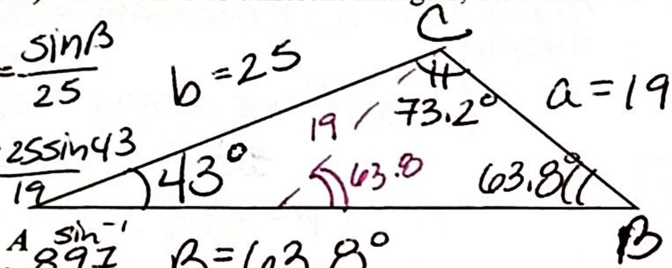


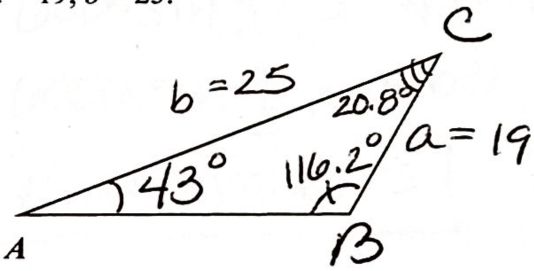
$\angle T = 180-(40+30)$
$= 180-70$
$\angle T = 110°$

$$\frac{\sin 110}{30} = \frac{\sin 40}{c}$$
$$\frac{c(\sin 110)}{\sin 110} = \frac{30\sin 40}{\sin 110}$$
$$\boxed{c = 20.5}$$

3a) Label the two different triangles, both with $<A = 43°$, $a = 19$, $b = 25$.

$$\frac{\sin 43}{19} = \frac{\sin B}{25}$$
$$\frac{19\sin B}{19} = \frac{25\sin 43}{19}$$
$$\sin^{-1} \sin B = .897 \sin^{-1}$$
$$B = 63.8°$$





3b) Solve the two triangles you sketched in part a. (all missing <'s & sides)

<B = $\boxed{63.8°}$

<C = $180-(43+63.8) = \boxed{73.2°}$

c = $\boxed{26.7}$

$$\frac{\sin 73.2}{c} = \frac{\sin 43}{19}$$
$$\frac{c\sin 43}{\sin 43} = \frac{19\sin 73.2}{\sin 43}$$

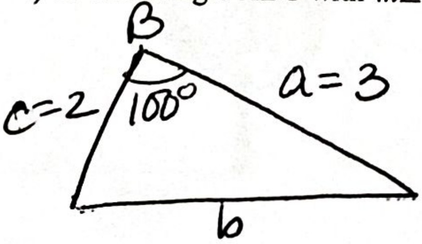
<B = $180-63.8 = \boxed{116.2°}$

<C = $180-(43+116.2) = \boxed{20.8°}$

c = $\boxed{9.9}$

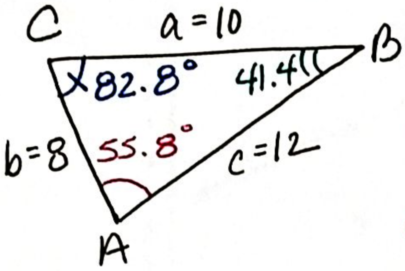
$$\frac{\sin 43}{19} = \frac{\sin 20.8}{c}$$
$$c\sin 43 = 19\sin 20.8$$
$$c = 9.9$$

4) Given triangle ABC with $m\angle B = 100°$, $c = 2$, $a = 3$, use Law of Cosines to solve for side b.



$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 3^2 + 2^2 - 2(3)(2)\cos 100°$$
$$b^2 = 15.08$$
$$\boxed{b = 3.88}$$

5) Given triangle ABC with $a = 10$ inches, $b = 8$ inches, and $c = 12$ inches, find the measure of its angles.



$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos A = \dfrac{8^2+12^2-10^2}{2(8)(12)}$

$\cos A = .5625$

$\boxed{A = 55.8°}$

$\dfrac{\sin 55.8}{10} = \dfrac{\sin B}{8}$

$\dfrac{10 \sin B}{10} = \dfrac{8 \sin 55.8}{10}$

$\sin B = .662$

$\boxed{B = 41.4°}$

$180 - (55.8 + 41.4)$

$\boxed{C = 82.8°}$

6) A forest has a rabbit population whose population fluctuates over time and can be modeled by the equation $y = 6{,}000 \sin \frac{\pi}{2}(x-4) + 10{,}000$ where y is the population of rabbits and x is the time after the population study begins in years.

a) Find the first time after the study begins when the population reaches 9,400 rabbits.

$9400 = 6000 \sin \frac{\pi}{2}(x-4) + 10000$

$-10{,}000 \qquad -10{,}000$

$\dfrac{-600}{6000} = \dfrac{6000 \sin \frac{\pi}{2}(x-4)}{6000}$

$\sin^{-1}(-.1) = \sin^{-1}\sin \frac{\pi}{2}(x-4)$

$\dfrac{-.1002}{\frac{\pi}{2}} = \dfrac{\frac{\pi}{2}(x-4)}{\frac{\pi}{2}}$

$-.0638 = x - 4$

$+4 \qquad +4$

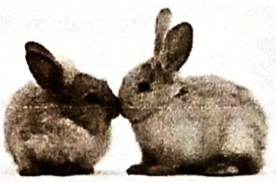
$\boxed{x = 3.94 \text{ years}}$

b) According to the model, what would the population be after 5 years? X

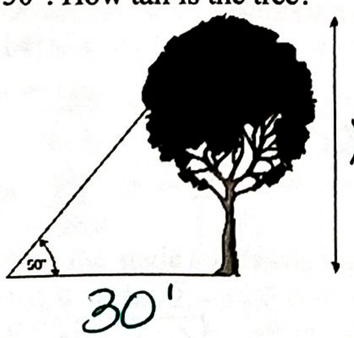
$y = 6000 \sin \frac{\pi}{2}(5-4) + 10000$

$y = 6000 \sin \frac{\pi}{2} + 10000$

$\boxed{y = 16{,}000 \text{ rabbits}}$



7) The angle of elevation from a spot on the ground 30 feet from the base of a tree to the top of a tree is 50°. How tall is the tree?



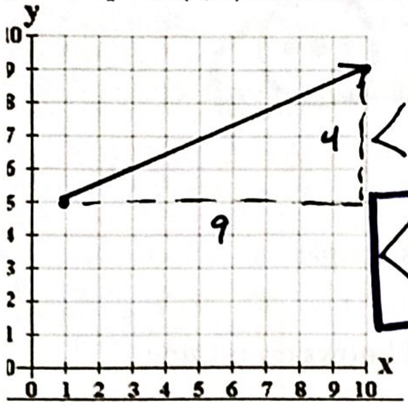
$\dfrac{\tan 50}{1} = \dfrac{x}{30}$

$x = 30 \tan 50$

$\boxed{x = 35.8 \text{ feet}}$

Find the component form for each vector.

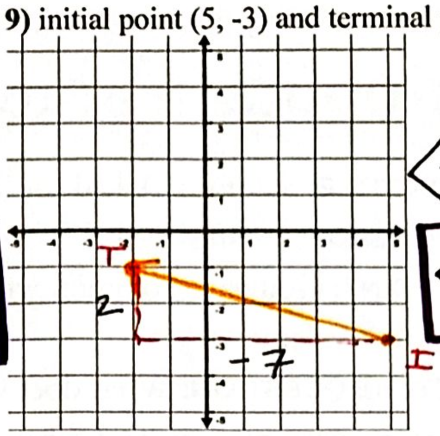
8) initial point (1, 5) and terminal point (10, 9)



$\langle 10-1, 9-5 \rangle$

$\boxed{\langle 9, 4 \rangle}$

9) initial point (5, -3) and terminal point (-2, -1)



$\langle -2-5, -1-(-3) \rangle$

$\boxed{\langle -7, 2 \rangle}$

10) Find the following given $\vec{\mathbf{u}} = \langle -1, -3 \rangle$, $\vec{\mathbf{v}} = \langle -3, 6 \rangle$

a) $\frac{2}{3}\vec{\mathbf{v}}$

$\frac{2}{3}\langle -3, 6 \rangle = \langle \frac{2}{3}(-3), \frac{2}{3}(6) \rangle$

$\boxed{\langle -2, 4 \rangle}$

b) $2\vec{\mathbf{v}} - 5\vec{\mathbf{u}}$

$2\langle -3, 6 \rangle - 5\langle -1, -3 \rangle$

$\langle -6, 12 \rangle + \langle 5, 15 \rangle = \boxed{\langle -1, 27 \rangle}$

11) Use the following vectors to complete the problems: $\vec{\mathbf{u}} = \langle 1, -4 \rangle$, $\vec{\mathbf{v}} = \langle 2, 5 \rangle$

a. $\vec{\mathbf{u}} + \vec{\mathbf{v}}$

$\langle 1, -4 \rangle + \langle 2, 5 \rangle$

$\boxed{\langle 3, 1 \rangle}$

b. $\vec{\mathbf{u}} - \vec{\mathbf{v}}$

$\langle 1, -4 \rangle - \langle 2, 5 \rangle$

$\langle 1-2, -4-5 \rangle$

$\boxed{\langle -1, -9 \rangle}$

c. $\|\vec{\mathbf{v}}\|$

$\|\vec{v}\| = \sqrt{2^2+5^2}$

$= \boxed{\sqrt{29}}$

≈ 5.4

Find the dot product of the following.

12) $\vec{\mathbf{u}} = \langle 0, -2 \rangle$ and $\vec{\mathbf{v}} = \langle 1, 10 \rangle$

$\vec{u} \cdot \vec{v} = 0(1) + (-2)(10)$

$= 0 + (-20)$

$= \boxed{-20}$

Find the value of **x** so that the vectors are orthogonal. $= 0$

13) $\vec{\mathbf{b}} = \langle \frac{5}{3}, -2 \rangle$ and $\vec{\mathbf{c}} = \langle x, 36 \rangle$

$\vec{b} \cdot \vec{c} = \frac{5}{3}x + (-2)(36)$

$0 = \frac{5}{3}x - 72$

$\left(\frac{3}{5}\right)72 = \frac{5}{3}x\left(\frac{3}{5}\right)$

$\boxed{x = 43.2}$

Determine if the vectors are **orthogonal, parallel,** or **neither.**

14) $\vec{\mathbf{u}} = \langle 39, -12 \rangle$, $\vec{\mathbf{v}} = \langle -26, 8 \rangle$

$m_{\vec{u}} = -\frac{12}{13} = -\frac{4}{13}$

$m_{\vec{v}} = \frac{8}{-26} = -\frac{4}{13}$

$\boxed{\text{parallel}}$

15) $\vec{\mathbf{u}} = \langle 8, -4 \rangle$, $\vec{\mathbf{v}} = \langle 5, 10 \rangle$

$\vec{u} \cdot \vec{v} = 8(5) + (-4)(10)$

$= 40 + (-40)$

0

$\boxed{\text{orthogonal}}$

Find the angle θ between the vectors.

16) $\vec{\mathbf{u}} = \langle 2\sqrt{2}, -4 \rangle$, $\vec{\mathbf{v}} = \langle -\sqrt{2}, 1 \rangle$

$\|\vec{u}\| = \sqrt{(2\sqrt{2})^2 + (-4)^2}$

$\|\vec{u}\| = \sqrt{24}$

$\|\vec{v}\| = \sqrt{(-\sqrt{2})^2 + 1^2}$

$\|\vec{v}\| = \sqrt{3}$

$\vec{u} \cdot \vec{v} = (2\sqrt{2})(-\sqrt{2}) + (-4)(1)$

$\vec{u} \cdot \vec{v} = -8$

$\cos\theta = \frac{-8}{(\sqrt{24})(\sqrt{3})}$

$\cos^{-1} \cos\theta = \cos^{-1} -.943$ $\boxed{\theta = 160.5°}$

17) $\vec{\mathbf{u}} = \langle 3, 1 \rangle$, $\vec{\mathbf{v}} = \langle 4, 5 \rangle$

$\|\vec{u}\| = \sqrt{3^2+1^2}$

$\|\vec{u}\| = \sqrt{10}$

$\|\vec{v}\| = \sqrt{4^2+5^2}$

$\|\vec{v}\| = \sqrt{41}$

$\vec{u} \cdot \vec{v} = 3(4) + 1(5) = 17$

$\cos\theta = \frac{17}{\sqrt{10}\sqrt{41}}$

$\cos^{-1} \cos\theta = \cos^{-1} .840$

$\boxed{\theta = 32.9°}$

18) Two soccer players kick a ball at the exact same time, the first player with a force of 25 N at a 60° angle in standard position. The second player kicks with a force of 30 N at an angle of 120° in standard position.

a) Write each force in component form.

$\vec{a} = \langle 25\cos 60, 25\sin 60 \rangle$

$\boxed{\vec{a} = \langle 12.5, 21.7 \rangle}$

$\vec{b} = \langle 30\cos 120, 30\sin 120 \rangle$

$\boxed{\vec{b} = \langle -15, 26 \rangle}$

b) Find the sum of the forces in component form.

$\vec{a}+\vec{b} = \langle 12.5+(-15), 21.7+26 \rangle$

$\boxed{\vec{a}+\vec{b} = \langle -2.5, 47.7 \rangle}$

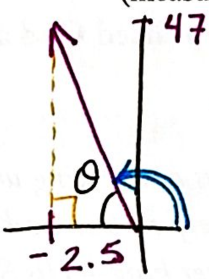
c) Find the magnitude of the combined forces.

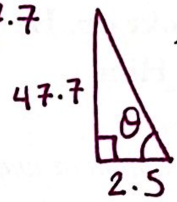
$\|\vec{a}+\vec{b}\| = \sqrt{(-2.5)^2+47.7^2}$

$= \sqrt{2281.54}$

$= \boxed{47.8N}$

d) Find the bearing of the resultant force. (measured from standard position)





$\tan\theta = \frac{47.7}{2.5}$

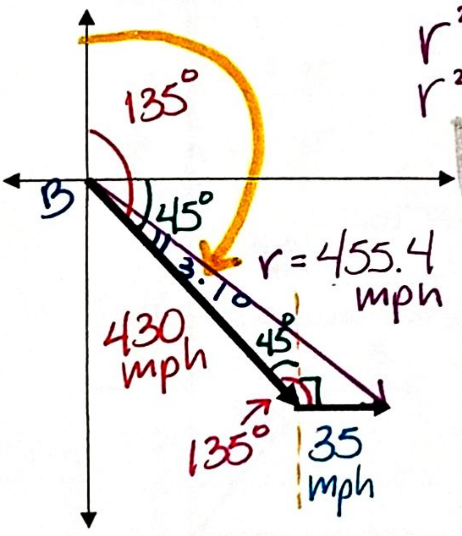
$\theta = \tan^{-1}\left(\frac{47.7}{2.5}\right)$

$\theta = 87°$

$180-87°$

$\boxed{\text{Bearing} = 93°}$

19) An airplane has an airspeed of 430 mph at a bearing of 135° (measured from due North.) The wind velocity is 35 mph blowing directly to the east. Find the resultant speed and bearing of the plane.



$r^2 = 430^2+35^2-2(430)(35)\cos 135$

$r^2 = 207,408.9$

$\boxed{r = 455.4 \text{ mph}}$ Resultant Speed

$\frac{\sin B}{35} = \frac{\sin 135}{455.4}$

$\frac{35\sin 135}{455.4} = \frac{455.4 \sin B}{455.4}$

$\sin B = .054$

$B = 3.1°$

Bearing =

$135 - 3.1°$

$\boxed{131.9° \text{ from due North}}$

Chapter 4

20) Change the degrees to radians and the radians to degrees. (leave as exact values)

a) $\frac{15}{24}\pi$ radians

$\frac{15\pi}{\cancel{24}_{2}} \cdot \frac{\cancel{180}^{15}}{\pi} = \boxed{112.5°}$

b) 115 degrees

$\cancel{115°}^{23} \cdot \frac{\pi}{\cancel{180}_{36}} = \boxed{\frac{23\pi}{36}}$

c) $\frac{4}{15}\pi$ radians

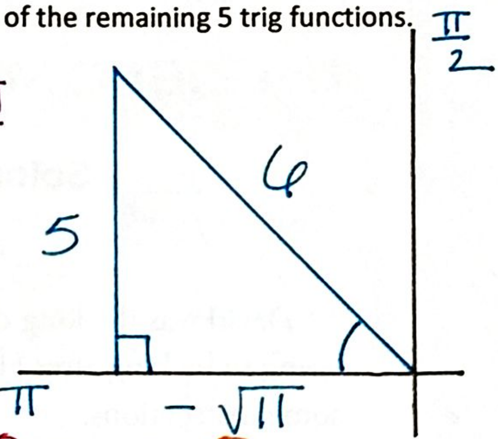
$\frac{4\cancel{\pi}}{\cancel{15}_{1}} \cdot \frac{\cancel{180}^{12}}{\cancel{\pi}} = \boxed{48°}$

d) -260 degrees

$-\cancel{260°}^{13} \cdot \frac{\pi}{\cancel{180}_{9}} = \boxed{\frac{-13\pi}{9}}$

21) If $\sin x = \frac{5}{6}, where... \frac{\pi}{2} \le x \le \pi$, find the exact value of each of the remaining 5 trig functions.

sin θ = $\frac{5}{6}$	csc θ = $\frac{6}{5}$	tan θ = $\frac{5}{-\sqrt{11}} = \frac{-5\sqrt{11}}{11}$
cos θ = $\frac{-\sqrt{11}}{6}$	sec θ = $\frac{-6}{\sqrt{11}} = \frac{-6\sqrt{11}}{11}$	cot θ = $\frac{-\sqrt{11}}{5}$



$x^2 + 5^2 = 6^2$

$x^2 = 11 \qquad x = \sqrt{11}$

22) Find the equation of a sine function shifted left 5, shifted up 9, with amplitude 4 and period $\frac{\pi}{2}$. p

$-h \qquad k \qquad a$

$pb = 2\pi$

$\frac{2}{\pi}\left(\frac{\pi}{2}b\right) = (2\pi)\frac{2}{\pi}$

$b = 4$

$$\boxed{y = 4\sin 4(x+5) + 9}$$

23) Find the equation of a cosine function shifted right 3, shifted down 9, with amplitude 7 and period π. p

$-h \qquad k \qquad a$

$pb = 2\pi$

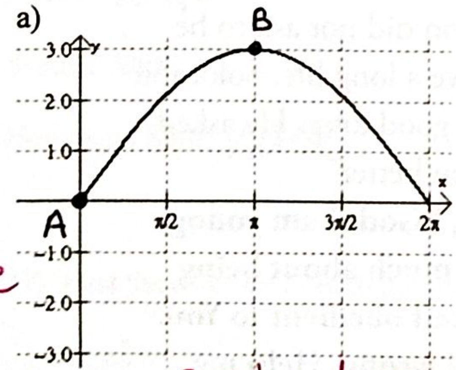
$\frac{\pi b}{\pi} = \frac{2\pi}{\pi}$

$b = 2$

$$\boxed{y = 7\cos 2(x-3) - 9}$$

24) Write an equation for each of the following sinusoidal curves using each indicated point:

a)



$b = \frac{1}{2}$

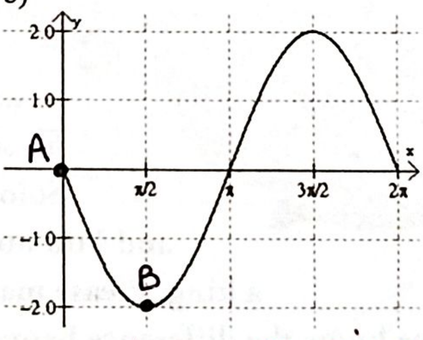
↑

half a cycle in 2π

A) $y = 3\sin \frac{1}{2}x$

B) $y = 3\cos\left(\frac{1}{2}(x - \pi)\right)$

b)



A) $y = -2\sin x$

B) $y = -2\cos\left(x - \frac{\pi}{2}\right)$

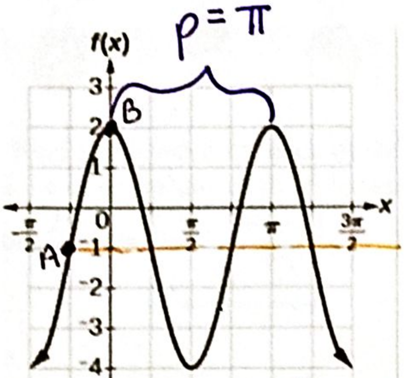
c)

$pb = 2\pi$

$\frac{\pi b}{\pi} = \frac{2\pi}{\pi}$

$b = 2$

$p = \pi$

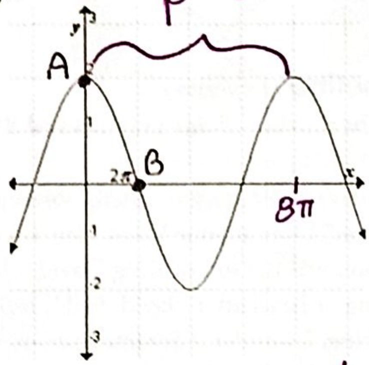


A) $y = 3\sin 2(x + \frac{\pi}{4}) - 1$

B) $y = 3\cos 2x - 1$

d)

$p = 8\pi$



$pb = 2\pi$

$\frac{8\pi b}{8\pi} = \frac{2\pi}{8\pi}$

$b = \frac{1}{4}$

A) $y = 2\cos \frac{1}{4}x$

B). $y = -2\sin \frac{1}{4}(x - 2\pi)$

For each of the following, state the amplitude, period and shifts, then graph 2 cycles of the curve.

25) $y = -4\cos(2x)$

Amplitude: -4 or 4

Period: π

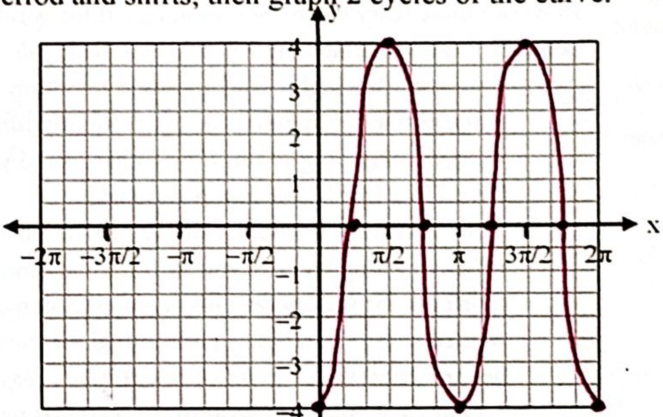
Vertical Shift: none

Horizontal Shift: none

$pb = 2\pi$

$p(2) = 2\pi$

$p = \pi$



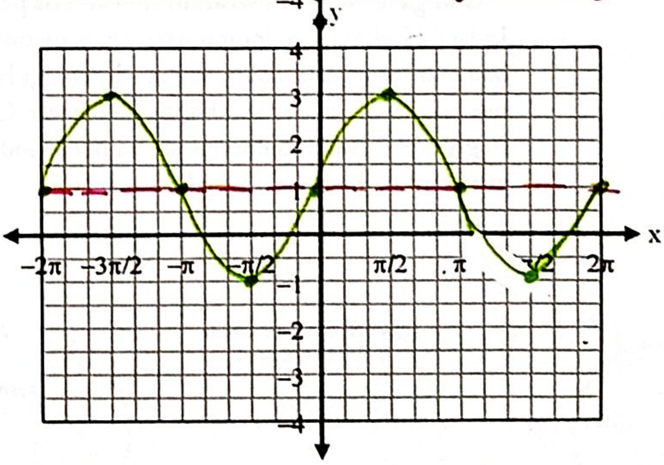
26) $y = 2\sin x + 1$

Amplitude: 2

Period: 2π

Vertical Shift: 1

Horizontal Shift: none



27) Find the measure of each missing side.

$\frac{\sin 34}{1} = \frac{x}{10}$

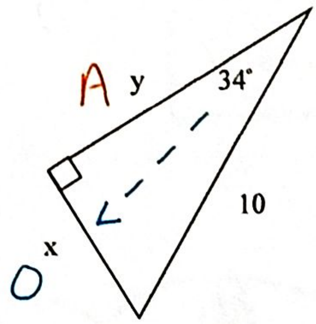
$x = 10\sin 34$

$\boxed{x = 5.6}$

$\frac{\cos 34}{1} = \frac{y}{10}$

$y = 10\cos 34$

$\boxed{y = 8.3}$



28) Find exact value for each of the following angles. Leave answers in simplified radical form.
Find the exact values of the following: (leave answers in simplified radical form)

a) $\tan\frac{2\pi}{3} = \boxed{-\sqrt{3}}$

b) $\sec\frac{3\pi}{2}$ $= \boxed{\text{undefined}}$

c) $\csc\frac{2\pi}{3} = \boxed{\frac{2\sqrt{3}}{3}}$

d) $\sin\pi = \boxed{0}$

e) $\cos\left(-\frac{5\pi}{4}\right) = \frac{3\pi}{4}$

$\cos\frac{3\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

f) $\cot\left(-\frac{11\pi}{6}\right) = \frac{\pi}{6}$

$\cot\frac{\pi}{6} = \boxed{\sqrt{3}}$

29) Using the idea of coterminal angles, find the exact value of the following:

a) $\sin\frac{11\pi}{2} - \frac{4\pi}{2} - \frac{4\pi}{2} = \frac{3\pi}{2}$

$\sin\frac{3\pi}{2} = \boxed{-1}$

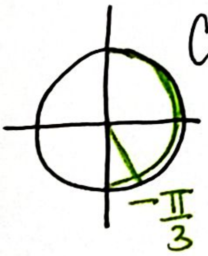
b) $\cos\frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$

$\cos\frac{5\pi}{6} = \boxed{-\frac{\sqrt{3}}{2}}$

Evaluate each expression:

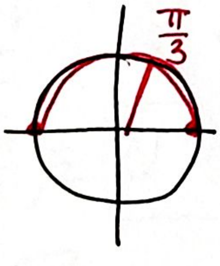
30) $\cos\left(\sin^{-1} -\frac{\sqrt{3}}{2}\right)$

$\cos\left(-\frac{\pi}{3}\right)$



$= \boxed{\frac{1}{2}}$

31) $\csc\left(\cos^{-1}\frac{1}{2}\right)$



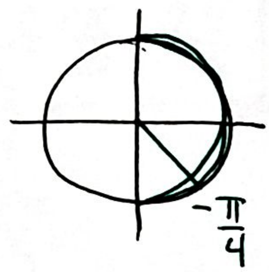
$\csc\left(\frac{\pi}{3}\right)$

$\boxed{\frac{2\sqrt{3}}{3}}$

32) $\sec(\arctan -1)$

$\sec\left(-\frac{\pi}{4}\right)$

$\boxed{\sqrt{2}}$



Chapter 5

Solve for θ on the interval of $[0, 2\pi)$

33) $\csc\theta = 2$

$$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$$

34) $\sec\theta = \sqrt{2}$

$$\boxed{\theta = \frac{\pi}{4}, \frac{7\pi}{4}}$$

35) $2\sin^2\theta - 1 = 0$

$$\frac{2\sin^2\theta}{2} = \frac{1}{2}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$

$$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

36) $\frac{3\tan^2\theta}{3} = \frac{1}{3}$

$$\sqrt{\tan^2\theta} = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3}$$

$$\tan\theta = \pm\frac{\sqrt{3}}{3}$$

$$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

37) $\cos^2\theta + 4\cos\theta + 3 = 0$

let $u = \cos\theta$

$$u^2 + 4u + 3 = 0$$

$$(u+1)(u+3) = 0$$

$$u + 1 = 0 \qquad u + 3 = 0$$

$$u = -1 \qquad u = -3$$

$$\cos\theta = -1 \qquad \cancel{\cos\theta} = -3$$

cos can't be smaller than −1

$$\boxed{\theta = \pi}$$

38) $\frac{\cos x \tan x}{\cos x} - \frac{\cos x}{\cos x} = 0$ GCF = cos x

$$\cos x(\tan x - 1) = 0$$

$$\cos x = 0 \qquad \tan x - 1 = 0$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \tan x = 1$$

$$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$$

39) $\sin 2\theta + 1 = 0$

let $u = 2\theta$

$$\sin u + 1 = 0$$

$$\sin u = -1$$

$$u = \frac{3\pi}{2}$$

$$\frac{1}{2}(2\theta) = \left(\frac{3\pi}{2}\right)\frac{1}{2} \qquad \boxed{\theta = \frac{3\pi}{4}}$$

40) $2\cos 4\theta - \sqrt{2} = 0$

let $u = 4\theta$

$$2\cos u - \sqrt{2} = 0$$

$$\frac{2\cos u}{2} = \frac{\sqrt{2}}{2}$$

$$\cos u = \frac{\sqrt{2}}{2}$$

$$u = \frac{\pi}{4} \qquad u = \frac{7\pi}{4}$$

$$\frac{1}{4}\,4\theta = \frac{\pi}{4}\cdot\frac{1}{4} \qquad \frac{1}{4}\,4\theta = \frac{7\pi}{4}\cdot\frac{1}{4}$$

$$\boxed{\theta = \frac{\pi}{16}, \frac{7\pi}{16}}$$

Simplify the following using identities:

41) $\cot\theta \bullet \sec\theta =$

$$\frac{\cancel{\cos\theta}}{\sin\theta}\cdot\frac{1}{\cancel{\cos\theta}} =$$

$$\frac{1}{\sin\theta} = \csc\theta$$

42) $\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta} =$ 1

$$\frac{1}{\sin\theta}$$

$$= \csc\theta$$

43) $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{\sec^2\theta}{\csc^2\theta}$

$$\frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} = \frac{1}{\cos^2\theta}\cdot\frac{\sin^2\theta}{1}$$

$$\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

Verify the following trigonometric identities:

44) $\dfrac{1}{\sin^3 x+\sin x\cos^2 x} = \csc x$

$$\frac{1}{\sin x(\underbrace{\sin^2 x+\cos^2 x}_{1})} =$$

$$\frac{1}{\sin x} =$$

$$\csc x = \checkmark$$

45) $\sec\theta - \sin\theta\tan\theta = \cos\theta$

$$\frac{1}{\cos\theta} - \sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) =$$

$$\frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta} =$$

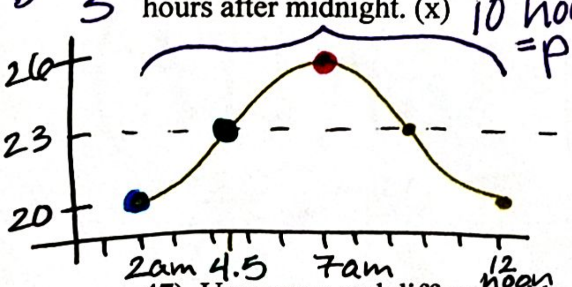
$$\frac{1-\sin^2\theta}{\cos\theta} =$$

$$\frac{\cos^2\theta}{\cos\theta} =$$

$$\cos\theta = \checkmark$$

$pb = 2\pi$
$10b = 2\pi$
$b = \frac{\pi}{5}$

46) At a dock in the San Francisco Bay, the depth of the water is 20 feet at low tide at 2am and high tide is 26 feet, which occurs 5 hours later. This motion can be modeled using a sinusoidal curve. Draw a graph and write a sinusoidal equation to represent the depth of the water (y) as a function of time in hours after midnight. (x) 10 hours $= p$



$$y = 3\cos\frac{\pi}{5}(x-7)+23$$

$$y = 3\sin\frac{\pi}{5}(x-4.5)+23$$

$$y = -3\cos\frac{\pi}{5}(x-2)+23$$

47) Use a sum and difference identity to find the exact value of sin 285°.

$$\sin(285) \Rightarrow \sin(150+135) = \underbrace{\sin 150}\underbrace{\cos 135} + \underbrace{\cos 150}\underbrace{\sin 135}$$

$$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{2}}{4} + -\frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$$

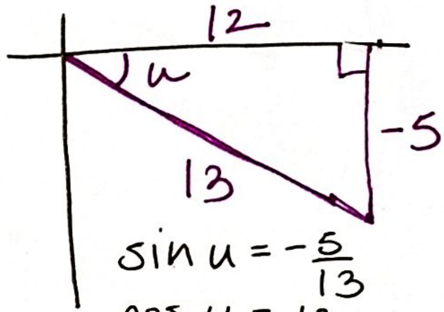
48) Use a sum or difference formula to find the exact value of cos 255°.

$$\cos 255 = \cos(210+45) = \underbrace{\cos 210}\,\underbrace{\cos 45} - \underbrace{\sin 210}\,\underbrace{\sin 45}$$

$$= \left(-\frac{\sqrt3}{2}\right)\left(\frac{\sqrt2}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt2}{2}\right)$$

$$= -\frac{\sqrt6}{4} - \left(-\frac{\sqrt2}{4}\right) = \boxed{\frac{-\sqrt6+\sqrt2}{4}}$$

If $\tan u = -\frac{5}{12}$ and $\frac{3\pi}{2} < u < 2\pi$, find the exact value of each of the following. (leave answers in simplified radical form when appropriate



$\sin u = -\frac{5}{13}$

$\cos u = \frac{12}{13}$

49) $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

$$+\sqrt{\frac{1-\frac{12}{13}}{2}}\,\frac{13}{13} = \sqrt{\frac{13-12}{26}}$$

$$= \sqrt{\frac{1}{26}} = \frac{\sqrt1}{\sqrt{26}} = \frac{1}{\sqrt{26}}\left(\frac{\sqrt{26}}{\sqrt{26}}\right) = \boxed{\frac{\sqrt{26}}{26}}$$

50) $\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$

$$-\sqrt{\frac{1+\frac{12}{13}}{2}} = -\sqrt{\frac{13+12}{26}}$$

$$= -\sqrt{\frac{25}{26}} = \frac{-\sqrt{25}}{\sqrt{26}} = \frac{-5}{\sqrt{26}} = \boxed{\frac{-5\sqrt{26}}{26}}$$

51) sin 2u $= 2\sin u\cos u$

$$2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right)$$

$$= \boxed{\frac{-120}{169}}$$

52) cos 2u $= \cos^2 u - \sin^2 u$

$$\left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$$

$$\frac{144}{169} - \frac{25}{169}$$

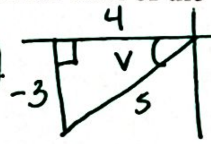
$$\boxed{\frac{119}{169}}$$

53) tan 2u $= \frac{2\tan u}{1-\tan^2 u}$

$$\frac{\cancel{2}\left(\frac{-5}{\cancel{12}\,6}\right)}{1-\left(\frac{-5}{12}\right)^2} = \frac{\left(\frac{-5}{\cancel{6}}\right)\cancel{144}\,24}{\left(1-\frac{25}{\cancel{144}}\right)144}$$

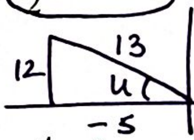
$$\frac{-120}{144-25} = \boxed{\frac{-120}{119}}$$

Find the exact value for each of the following given $\cos u = -\frac{5}{13}$ where $\frac{\pi}{2} < u < \pi$, and $\sin v = -\frac{3}{5}$ where $\pi < v < \frac{3\pi}{2}$ QIII



$\sin v = -\frac{3}{5}$

$\cos v = 4/5$ $\quad \tan v = -\frac{3}{4}$



$\cos u = \frac{-5}{13}$ $\quad \tan u = \frac{-12}{5}$

$\sin u = \frac{12}{13}$

54) $\cos(u-v)$

$$= \cos u\cos v + \sin u\sin v$$

$$\left(\frac{-5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right)$$

$$\frac{-20}{65} + \frac{-36}{65} = \boxed{\frac{-56}{65}}$$

55) $\tan(u+v)$

$$\frac{\tan u + \tan v}{1-\tan u\tan v}$$

$$= \frac{\left(\frac{-12}{5}\right)+\left(\frac{-3}{4}\right)}{1-\left(\frac{-12}{5}\right)\left(\frac{-3}{4}\right)}$$

$$\frac{\overset{4}{\cancel{20}}\left(\frac{-12}{\cancel{5}} - \frac{3}{\cancel{4}}\right)\overset{5}{\cancel{20}}}{20\left(1-\frac{36}{\cancel{20}}\right)\cancel{20}}$$

$$\frac{-48-15}{20-36} = \frac{-63}{-16}$$

$$= \boxed{\frac{63}{16}}$$