Trigonometry/Precalculus
Logarithmic Application Problems
Day _____

Name
Date
Block

1) There once was an old man named Mr. Long who had a terrible case of rheumatoid arthritis. Mr. Long's doctor, Dr. Short, tried many progressive medicines but could not come up with anything better than the old standby of aspirin. The good doctor told Mr. Long that the only way to effectively treat the arthritis was to take large doses of aspirin. The concentration of the aspirin in the bloodstream is given by an exponential function of the form $y = a \cdot e^{kx}$ where y is the concentration of aspirin (in mg per cubic centimeter of blood) and x is the time in hours after taking the dose of aspirin. At the end of one hour after taking the aspirin, Dr. Short measured the concentration of aspirin in Mr. Long's bloodstream to be 12.539 mg per cubic centimeters and at the end of four hours the amount of aspirin was 7.759 mg per cubic centimeters.

a) Find the equation that would approximate the amount of aspirin in the bloodstream for any time $t \geq 0$.

$y = 14.715e^{-.16x}$

b) Find out what the original dose that Dr. Short gave to Mr. Long.

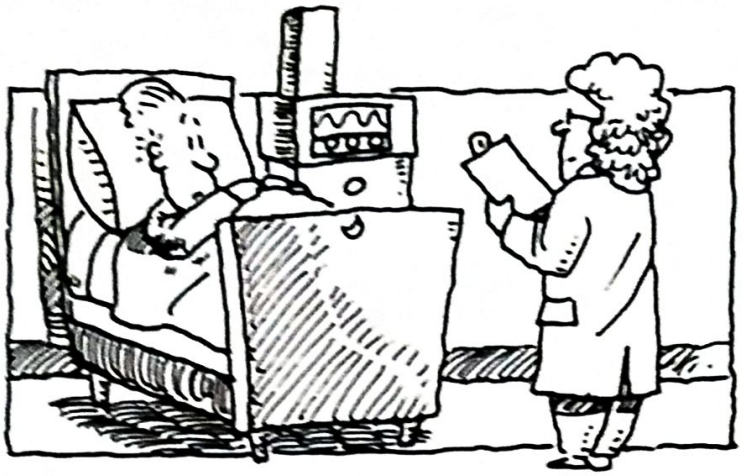
$a = 14.715$

c) Mr. Long says that he starts feeling the pain around six and a half hours. How much aspirin is still in his bloodstream when he feels the pain?

$y = 5.2 \text{ mg/cm}^3$

d) When there is 3 mg/cm^3 left, the pain becomes unbearable to the point that another dose is prescribed. Find the approximate time when another dose needs to be taken.

$x = 9.94$ hours



2) Ginnie Gerbil is worried about Hollywood performer George Gerbil. George has developed a tumor in his ear that appears to be growing. A mistake occurred in a laboratory experiment and he was given a cancer-causing agent that tests the growth rate of the cancer. Dr. John has been monitoring George since the tragic experiment and found that on the first day after the injection the size of the tumor was 0.322 cubic centimeters and on the third day it was measured as 0.426 cm^3.

a) If the size of the tumor is approximated by an exponential function of the form $y = a \cdot e^{kx}$, where y is the size of the tumor and x is the number of days since the injection that caused the tumor, find an equation that could approximate the size of the tumor.

$y = .28e^{.14x}$

b) If the tumor reaches three-fourths of a cubic centimeter, George will have to have his ear cut off and will no longer be able to participate in Hollywood productions. The good thing is that if George can get radiation therapy every day, he can reverse the growth of the tumor and completely eradicate it. The only place in Hollywood to get the radiation therapy is booked and George cannot get in for seven days after he was injected. Help Ginnie decide if she needs to worry or if can she be confident the radiation therapy will eliminate the tumor in time.

$x = 7$

$y = .7461\ cm^3$

c) George cannot perform if the size of the tumor is larger than 0.5 cm^3. Find out how long George can perform until the tumor gets too big.

$y = 4.14$ days

4 days, 3 hours, 22 minutes

3) Congratulations! Your rich old Uncle Lew has bestowed $10,000 upon you. The only problem is, that in order to receive the money, you have to decide which account is more beneficial to place the money in over the course of one year. If you choose correctly, you get to keep the money as a part of your college education. If you choose incorrectly, the money goes to help needy children get through their math courses. The choice of accounts are 5% compounded daily (365 days) or the same 5% compounded continuously. Which account would be the most beneficial if you left the money in it for one year?

Continuously $A = Pe^{rt}$

Interval $A = P\left(1+\frac{r}{n}\right)^{nt}$

4) The Richter scale was proposed in 1935 by Charles Richter and refined in 1979. The Richter scale measures the severity of an earthquake. The severity can be measured either by the amplitude of the seismic wave or by the energy released. A one-point increase in the Richter scale umber corresponds to a ten-fold increase in the amplitude of the seismic wave.

a) The energy released is proportional to the $\frac{3}{2}$ power of the amplitude. Show that this implies that a one-point increase in the Richter scale number corresponds to about a thirty-one-fold increase in the energy released.

$10^{3/2} = 31.623$

b) One form of Richter's equation is $M = \frac{2}{3}\log E + k$, where M is the magnitude of the quake on the Richter scale, E is the energy of the earthquake in joules, and k is a constant. If a quake which releases 10^{13} joules of energy has a Richter magnitude of 5.8, solve for the constant k and rewrite Richter's equation.

$5.8 = \frac{2}{3}\log 10^{13} + k$

$5.8 = \frac{2}{3}(13) + k$

$5.8 = \frac{26}{3} + k$

$K = -2.87$

$M = \frac{2}{3}\log E - 2.87$

c) The great San Francisco earthquake of 1906 had what was estimated to be somewhere between 7.9 and 8.3 on the Richter scale, and moved one section of ground in Olema 20 feet. Assuming the earthquake measured 8.3, how many joules of energy were released?

$E = 5.69 \times 10^{16}$ joules

d) The 1989 Loma Prieta earthquake was about 7.1 on the Richter scale. How many times as much energy was released by the great San Francisco quake of 1906?

$7.1 = \frac{2}{3}\log E - 2.87$

$\frac{E_{8.3}}{E_{7.1}} =$ 63.1 times more!

5) 7) A biologist researching a newly-discovered species of bacteria, whose growth can be measured by the equation $A = Pe^{kt}$. At time t = 0 hours, he puts 100 bacteria into what he has determined to be a favorable growth medium. Six hours later, he measures 450 bacteria.

a) Assuming exponential grown, what is the growth constant "k" for the bacteria?

$k = .25$

$A = 100e^{.25t}$

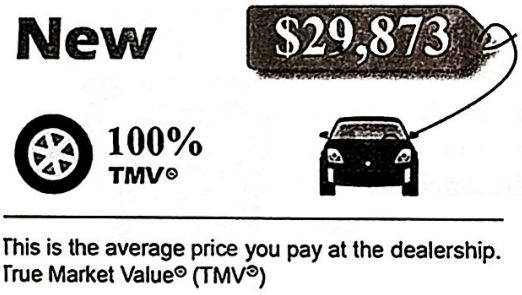
b) How many bacteria will there be after 12 hours?

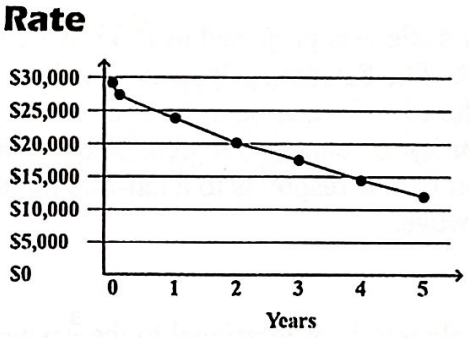
2009 bacteria

c) How long will it take for the original sample to grow to 1000 bacteria?

$t = 9.2$ hours

6) The depreciation of a new car can be represented by an exponential curve. The following curve represents the average cost of a new car and its worth over time.





a) Using the equation $y = ab^x$, where a is the initial value, b is the rate of depreciation, x is time in years, and y is current value, find the equation that represents the depreciation if after 5 years the new car's value is $12,069.

$y = 29{,}873(.83)^x$

b) According to this equation, what will the car's value be after 8 years?

$y = \$6{,}728.27$

c) In how many years will the car's value be $5,000?

$x = 9.6$ years

Log Review ~ Solve each of the following for x.

7) $\frac{1}{2}\log x + \log 4 = \log 20$

$x = 25$

8) $\log(3x + 8) - \log(x - 4) = \log 5$

$x = 14$

9) $8^{x-1} = 4^{2x+7}$

$x = -17$