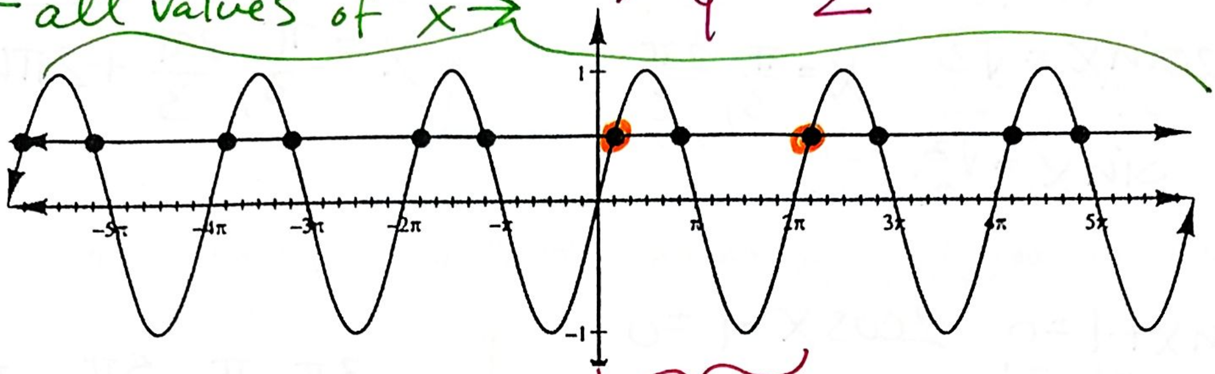
Solving Trigonometric Equations Notes

$\sin x = \frac{1}{2}$ → $y = \sin x$

→ $y = \frac{1}{2}$

← all values of x →



$[0, 2\pi]$

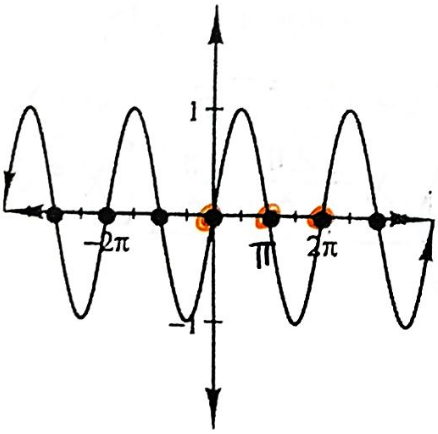
for x-values between $[0, 2\pi]$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

for all values of x.

$x = \frac{\pi}{6} + 2\pi n$

$x = \frac{5\pi}{6} + 2\pi n$



$\sin x = 0$

for x-values between $[0, 2\pi]$

$\boxed{x = 0, \pi, 2\pi}$

for all values of x.

$\boxed{x = 0, \pi, 2\pi + 2\pi n}$

$= \boxed{x = \pi n}$

Example: Solve for x in the indicated domain. When no domain is given, find all values of x.

a) $2\sin x - \sqrt{3} = 0$, *for the domain* $[0, 2\pi]$

$+\sqrt{3} \quad +\sqrt{3}$

$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

b) $2\sin x - \sqrt{3} = 0$, *for all x*

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3} + 2\pi n}$

c) $(\sin x + 1)(2\cos x - 1) = 0$, *for the domain* $[0, 2\pi]$

$\sin x + 1 = 0 \qquad 2\cos x - 1 = 0$

$-1 \quad -1$

$\sin x = -1 \qquad 2\cos x = 1$

$\cos x = \frac{1}{2}$

$\boxed{x = \frac{3\pi}{2}} \qquad \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

d) $(\sin x + 1)(2\cos x - 1) = 0$

$\boxed{x = \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3} + 2\pi n}$

Notation note:

$(\sin x)^2 = \sin^2 x$

$\sin^2 x \neq \sin x^2$

$u = \cos x$

e) Solve $2\cos^2 x - \cos x - 1 = 0$ for all values of x in the domain $[0, 2\pi)$

$2u^2 - u - 1 = 0$

$(2u + 1)(u - 1) = 0$

$-2u \qquad u$

$2u + 1 = 0 \qquad u - 1 = 0$

$u = -\frac{1}{2} \qquad u = 1$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}} \qquad \boxed{x = 0}$

f) Solve $\sin^2 x - 1 = 0$ or all values of x.

$+1 \quad +1$

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$

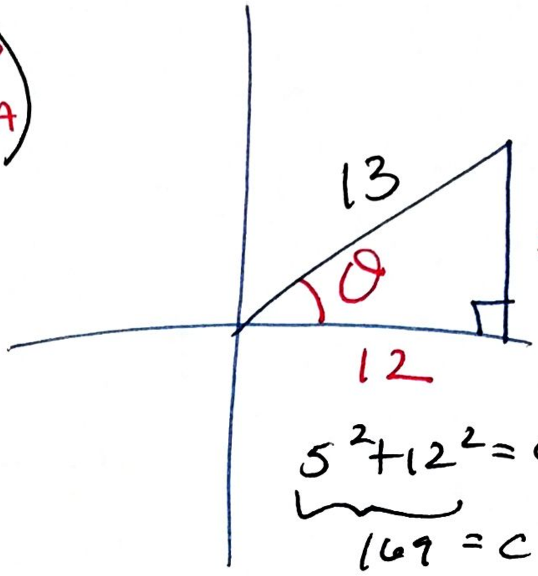
$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2} + 2\pi n}$

4.7 notes cont.

ex:

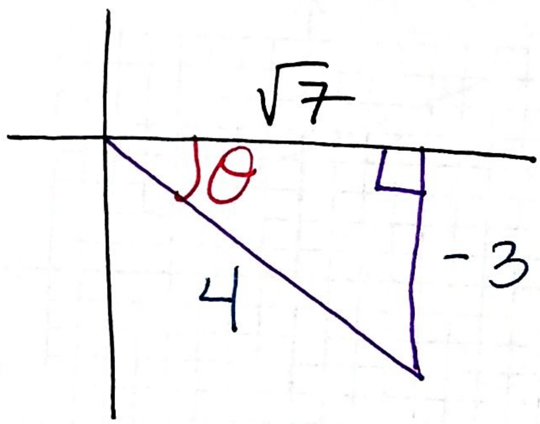
$$\cos\left(\underbrace{\tan^{-1}\frac{5\ \text{O}}{12\ \text{A}}}_{\theta}\right)$$

$$\cos\theta = \boxed{\frac{12}{13}}$$



$$\underbrace{5^2+12^2}=c^2$$
$$169=c^2$$
$$c=13$$

ex: $\tan\left[\underbrace{\arcsin -\frac{3}{4}}_{\theta}\right]$



$$\tan\theta = \frac{-3}{\sqrt{7}}\left(\frac{\sqrt{7}}{\sqrt{7}}\right)$$

$$= \boxed{\frac{-3\sqrt{7}}{7}}$$

$$(-3)^2+x^2=4^2$$
$$9+x^2=16$$
$$\sqrt{x^2}=\sqrt{7}$$
$$x=\sqrt{7}$$