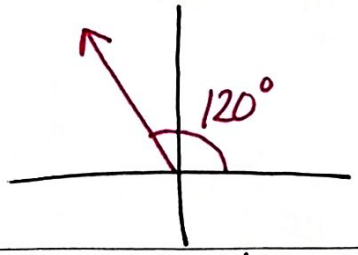
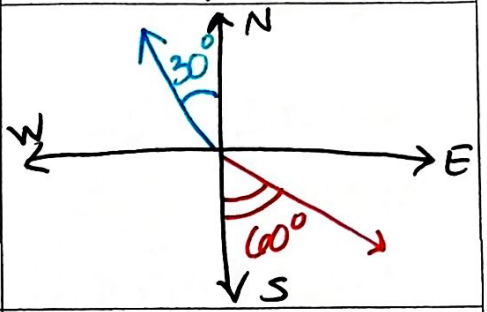
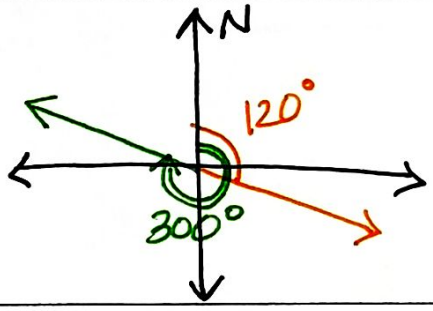
Vector Application Using Law of Cosines Notes

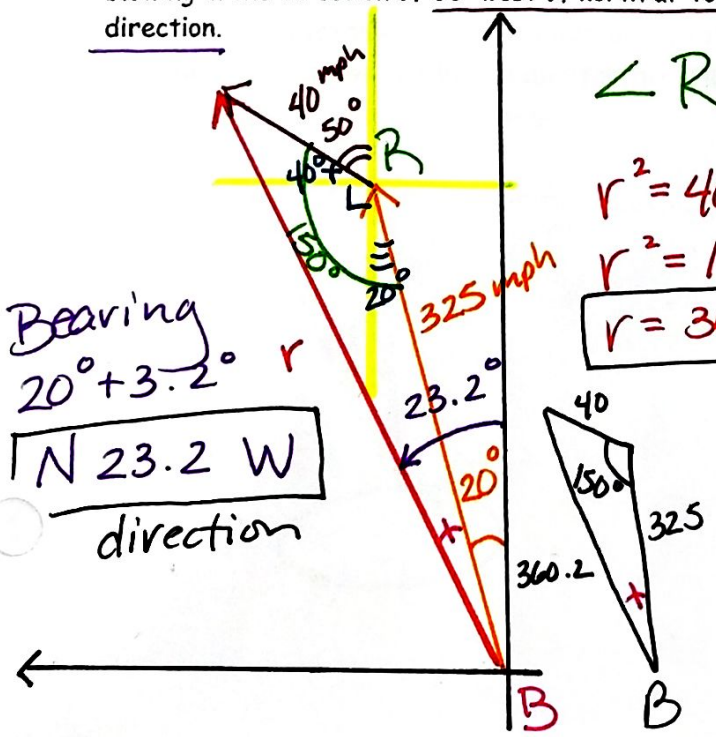
Vector Directions can be given 3 different ways:

FORM	PHRASING	SKETCH
Standard Form (ccw from positive x-axis)	An angle formed with the positive x-axis.	
Bearing N ___° E/W S ___° E/W	Bearing of... N 30° W S 60° E	
Bearing CW from North	Bearing of 120° Bearing of 300°	

Though finding a resultant vector is always possible by writing vectors in component form, using the Law of Cosines can sometimes be a faster method to find magnitude and direction of a resultant vector.

↳ tail-to-tip method

Ex: A plane is traveling in the direction of 20° west of north at a speed of 325 mph. A wind is blowing in the direction of 50° west of north at 40 mph. Find the plane's resultant velocity and direction.



$\angle R = 40° + 90° + 20° = \boxed{150°}$

$r^2 = 40^2 + 325^2 - 2(40)(325)\cos 150°$

$r^2 = 129{,}741.7$

$\boxed{r = 360.2 \text{ mph}}$ Resultant Velocity

$\frac{\sin 150}{360.2} = \frac{\sin B}{40}$

$\frac{40 \sin 150}{360.2} = \frac{360.2 \sin B}{360.2}$

$\sin B = .055$

$\boxed{B = 3.2°}$

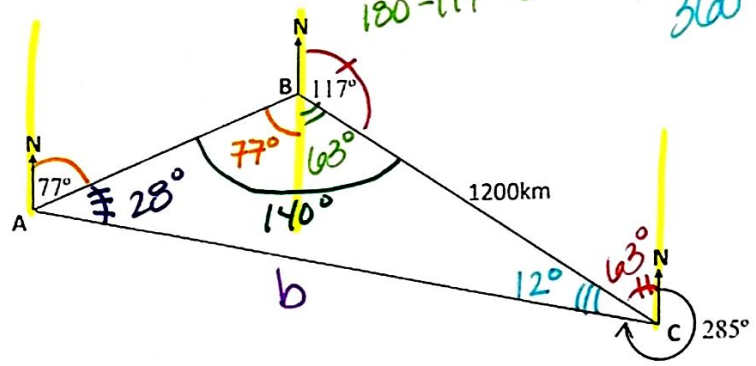
Bearing

$20° + 3.2°$

$\boxed{N\ 23.2\ W}$

direction

Ex: A captain planning the course for a voyage knows that he has to sail from Port A to Port B with a bearing of 77° (from due North) and from there to Port C with a bearing of 117° for 1200 km. In order to return to port A, the ship has to sail with a bearing of 285°. How far would he have to sail to return from port C to port A?



$180 - 117 = 63°$

$360 - (63 + 285) = 12°$

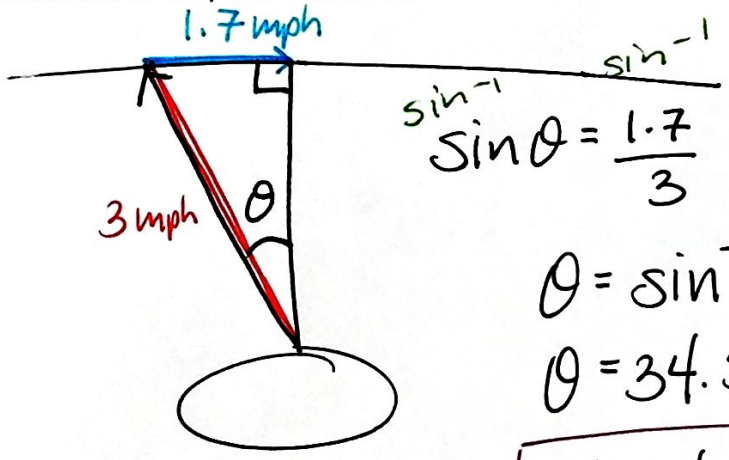
$180 - (140 + 12) = 28°$

$$\frac{\sin 140}{b} = \frac{\sin 28}{1200}$$

$$\frac{b \sin 28}{\sin 28} = \frac{1200 \sin 140}{\sin 28}$$

$$\boxed{b = 1643 \text{ km}}$$

Ex: The Sharkfest is a swim from Alcatraz to Crissy Field in San Francisco. Though their target is directly in front of them, swimmers are told to aim to the left of their target because of the 1.7 mph ebb tide current that flows toward the right. If someone swims with an average speed of 3 mph, what angle should they swim at (West of North) in order to end up directly at the finish on the beach directly across from them?



$\sin^{-1}$ $\sin^{-1}$

$$\sin\theta = \frac{1.7}{3}$$

$$\theta = \sin^{-1}\left(\frac{1.7}{3}\right)$$

$$\theta = 34.5°$$

$$\boxed{N\ 34.5°\ W}$$